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Forest Inventories

Discrepancies and Uncertainties

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Forest Inventories: Discrepancies and Uncertainties

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Abstract

Credits for sequestered carbon augment forests' already considerable value as natural habitat and as producers of timber and biomass, making their accurate inventory more critical than ever before. This article examines discrepancies in inventories of forest attributes and their sources in four variables: area, timber volume per area, biomass per timber volume, and carbon concentration. Documented discrepancies range up to a multibillion-ton difference in the global stock of carbon in trees. Because the variables are multiplied together to estimate an attribute like carbon stock, more precise measurement of the most certain variable improves accuracy little, and a 10 percent error in biomass per timber levers a discrepancy as much as a mistake in millions of hectares. More precise measurements of, say, accessible stands cannot remedy inaccuracies from biased sampling of regional forests. The discrepancies and uncertainties documented here underscore the obligation to improve monitoring of global forests.

Key Words: forest monitoring, Forest Identity, forest carbon, remote sensing

JEL Classification Numbers: Q23, Q56, Q57

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Forest Inventories: Discrepancies and Uncertainties

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Introduction

Despite the economic and environmental significance of forests, we have only imprecise measurements of the physical variables that determine their valued attributes, whether for natural habitat, timber volume, biomass, or stored carbon. Without accuracy, appraisals of timber will be discredited, assays of biomass will be deceptive, and claims of sequestered carbon may be fraudulent. This article illustrates discrepancies and the sources of uncertainty to document flaws in the foundation of forest measurements.

Section I provides examples of discrepancies in forest inventories. Section II uses an organizing principle, the Forest Identity (set forth in Kauppi et al. 2006 and Waggoner 2008), to show how these discrepancies confound estimates of the physical variables of forest area, timber density, biomass per timber volume, and carbon concentration. Generally, area is measured as thousand hectares (kha), volume as cubic meters (m^3), mass as Mg (ton), and timber density as volume of growing stock per hectare (m^3/ha). The Forest Identity exposes how uncertainties about constituent variables like timber density propagate into uncertainties about attributes like national carbon stock. Section III integrates the uncertainties of the variables into uncertainties of attributes. Integrating uncertainties includes three parts: the estimate itself, a margin of error, and the probability that actually lies within the margin. Section IV illustrates a fourth part, bias. Section V concludes.

I. Discrepancies, Illustrated

Eleven discrepancies, ranging from tropical forest area to global carbon stock, illustrate uncertain measurement and monitoring.

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1. How many hectares of the world did tropical forests occupy in 1990?

Although tropical deforestation is a prominent concern, establishing a reference area to calculate change in the area of tropical forests and thus a basis for the concern proves difficult. Consider establishing a reference area: the Food and Agriculture Organization's 1990 forest resource assessments (FRA) in 90 tropical lands reported first 1,756 million ha, then 1,925 million ha, and finally 1,949 million ha (Grainger 2008). Thus, estimates by the same authoritative international organization for the same year shifted first by 10 percent and then an additional 1 percent to become an 11 percent discrepancy in a reference area for calculating deforestation.

2. How much Amazonian forest have Brazilians cleared?

The vast area of Brazilian forests yields impressive cases of deforestation. Four estimates of the deforestation of Brazil or parts of it from 2000 to 2005 appear in Table 1. The area of all land, the forest area in 2000, and the deforested area during the five years lead to the final column, the percentage cleared in five years.

FRA 2005 reported that from 2000 to 2005, Brazilian forests shrank by 15,515 kha (row 1); this accounted for 3.2 percent of the FRA 2005 report of national forest area in 2000.

Using remote sensing, Hansen et al. (2008) estimated that tropical deforestation in the nation of Brazil was 2,600 kha/yr during 2000 to 2005, which they contrasted with the national FRA 2005 rate of 3,100 kha/yr. "For Brazil …, the FRA reports annual change in forest area from 2000 to 2005 equal to 3.10 … million ha/yr … Our estimates of forest clearing for Brazil … are 2.60 … million ha/yr." Five times 2,600 equals 13,000 kha in five years (row 2).

Brazil's PRODES reported that annual deforestation in the nine states of Amazonia from 2000 to 2005 totaled 11,070 kha, which was 3.3 percent of the 335,349-kha forest in 2000 (row 3). For a description of PRODES, see Earth Observation Coordination (2008).

Hansen et al. remotely sensed humid tropical deforestation of 2,600 kha/yr during the five years, which totals 13,000 kha (row 4). They stated that deforestation for the five years was 3.6 percent, which equals the five-year loss divided by 361,160 kha.

The estimated rates of national, Amazonian, and humid tropical deforestation during the five years all lie between 2 and 4 percent. These rates are of course alternative estimates, but here and elsewhere it is understood that they are estimates and not absolute truth or facts. For all Brazil, FRA 2005 reported 19 and 40 percent more deforestation than the PRODES and Hansen estimates of Amazonian deforestation. Hansen estimated a 17 percent greater loss from humid tropical forest than the Brazilian estimate for a similar area in nine Amazonian states.

3. In a smaller nation, Costa Rica, do estimates of forest area agree?

Costa Rica is another tropical nation but with only 5,000 kha—a much more comprehensible expanse than Brazil with its vast forests.

A less stringent test of accuracy than knowing the precise number of hectares of forest would be simply knowing whether forests had begun expanding again after a period of deforestation—a reversal from net deforestation to net expansion that is called the forest transition (Mather et al. 1999). Unfortunately, surveys failed even this less stringent test in Costa Rica, as well as in six of seven other nations, during the quarter-century ending in 2005. Table 2 shows the different narratives that Grainger (2009) derived from three methods for identifying the forest transition. The FAO forest resource assessments (column 1) showed a certain or possible reversal of deforestation in Costa Rica and two other nations. The reports that nations sent to FAO (column 2) indicate instead a steady rise in deforestation in Costa Rica. In six of seven other nations, the evidence sent to FAO disagreed with the assessments published by FAO. The pattern in time series of actual surveys (column 3), recognized as valid by the government in its reports to FAO, indicates that the direction of change in Costa Rica was uncertain. The direction of change in six of the seven other nations, assayed by the method in column 3, disagreed with the results from the other two methods. Grainger attributed these discrepancies to simultaneous deforestation and reforestation in different regions of a nation, and to the differing perceptions among analysts focusing on the global scale rather than national scale.

Colleagues with 20 years' experience in Costa Rica distilled their experience in two reports, Sanchez-Azofeifa et al. (2009) and Kalacska et al. (2008). They compiled a reference map or baseline, found that it correctly identified forest versus nonforest 90 to 92 percent of the time at 800 points, and produced a reference value of 2,323 kha of Costa Rican forest in 2000. FRA 2005 reported nearly the same area for 2000. Four remotely sensed data sets, all with spatial resolutions of 100 ha, produced estimates of 20 to 70 percent more forest, while a coarse-resolution remote sensing in 1998 estimated 42 percent less (Kalacska et al. 2008, Table 3). Sanchez-Azofeifa (2009, Table 2) calculated that MODIS remote sensing with 250-by-250m or 500-by-500m resolution would have detected 19 or 29 percent less forest, respectively, during 1987 to 1997 than Landsat, with 30- by-30m resolution. For *deforestation,* the MODIS estimated fully 84 or 94 percent less deforestation than Landsat did.

The Geo-Wiki Project displays differences among three surveys of forest and asks volunteers to review disagreement hotspots on maps of global land cover (Geo-Wiki 2009). The volunteers use their local knowledge to determine whether the land cover is correctly mapped. Several areas in Costa Rica and throughout Central America and Mexico show disagreement between MODIS and the Global Land Cover 2000 map used by the Millennium Ecosystem Assessment (Bartholomé and Belward 2005). The panels of Figure 1 show three maps: the GLC 2000 classification of land use (lower left), the MODIS classification of land use (lower right), and the disagreement about forest cover between the two (upper panel). In many areas, the MODIS classification indicates 5 to 40 percent more forest, and in a few areas, 40 to 60 percent more forest than GLC 2000. The red circle in Figure 1 identifies a hotspot of disagreement north of Costa Rica in El Salvador and Guatemala and thus a priority place for volunteers to correctly identify forest area.

The colleagues in the Costa Rican surveys (Kalacska et al. 2008, 355) concluded by warning,

> Variations in baseline estimations may end up costing millions of misspent dollars over time and frustration in both the governmental and scientific spheres of political action. As indicated … the relatively large underestimation of the baseline by the land cover maps for the Costa Rican example leads to compounded errors in forecasted carbon sequestration, and it is unrealistic and erroneous to assume that if the rate of change is correct that the initial baselines are not important.

4. Are Salvadoran forests shrinking or expanding?

According to the 2005 forest resource assessment (FAO 2005), the forested area in El Salvador shrank 14 percent from 1990 to 2000 and then another 8 percent by 2005. Based on satellite images, however, Hecht and Saatchi (2007) reported that, from the early 1990s to the early 2000s, El Salvadoran land with more than 30 percent tree cover expanded 22 percent, and the area of denser forests, with more than 60 percent tree cover, expanded 25 percent: “The overall area of woodland increased in the 1990s, with the main pattern of woodland resurgence occurring in relatively small patches as crop and grazing lands reverted to woodlands throughout the country.” The two appraisals differ in more than quantity; they even differ in direction—shrinkage versus expansion. The discrepancy illustrates the problem of whether to call a group of trees a *forest*.

5. Do estimators sometimes assume that forest density never changes?

For the 50 nations with largest forests, Kauppi et al. (2006) mapped the changing forest area and density of growing stock per ha on a synoptic chart (Figure 2). Several nations lined up along the equator of unchanging forest density. The 19 nations within the oval on Figure 2 reported the same forest density of growing stock per ha in 1990 and 2005 (Angola, Bolivia, Brazil, Cameroon, Canada, Central African, Congo, DR Congo, French Guiana, Gabon, Guatemala, Lao, Madagascar, Mongolia, Romania, South Africa, Sudan, Suriname, Zambia). Among the 50 nations with the greatest timber volumes, 4 nations reported exactly the same area in 1990 and 15 years later, in 2005 (Romania, Canada, South Africa, Suriname). A report of identical estimates of forest parameters for 15 years strains belief.

6. Do IPCC methods agree with each other?

Sensitive to decades of environmental research and outcry about carbon released by deforestation, the Intergovernmental Panel on Climate Change formulated procedures for estimating carbon (IPCC 2003). Brown et al. (2007) examined the quantification of reduction of carbon emissions from deforestation and degradation (REDD) in developing countries and compared two estimates of forest biomass. The first method, called IPCC Tier 1, proceeds from mean annual increments of trees in broad forest types, such as African tropical rainforest. The increments are tabulated in the IPCC emission factor database (EFDB). Another IPCC method, "Good Practice," employs plots. In Brown's six comparisons (Table 3), Tier 1 overestimated carbon density as much as 33 percent in a Mexican temperate forest and underestimated density as much as 44 percent in an African rainforest.

7. Do formulae to estimate the biomass in a single place agree?

In 2003 Smith (2003, Section 3.4) and others surveyed a Zambian wilderness area that receives about 1,000 mm annual rainfall. They used two equations to convert their observations of trees into biomass and wrote,

> A hectare of miombo woodland in the Mutinondo Wilderness Area is measured as containing 37.366 [Mg] of biomass, using the Sengwa formula. This is equivalent to 18.683 [Mg] of carbon per hectare. Using the central Zambian formula of Chidumayo, a hectare of MWA miombo woodland contains 132.23 [Mg] of biomass, equivalent to 66.115 [Mg] of carbon.

If carbon estimates for a single forested hectare show a threefold difference, discrepancies among estimates for larger areas will not surprise.

8. How fast are intact forests growing in Africa?

From measured diameters of trees on 79 permanent plots in moist tropical African forests, Lewis et al. (2009) estimated biomass and carbon. From annual changes during 1968 to 2007 that encompassed losses of 6 and gains of 4 Mg of carbon per year, they calculated an average gain of 0.63. They then calculated a weighted average of 202 Mg C/ha above ground in live trees—an average that far exceeds Smith's (2003) estimates of 19 to 66 Mg C/per ha in Zambian woodlands but lies within the range of 148 to 277 Mg/ha in African, Brazilian, and Indonesian tropical rainforests (Table 3). To estimate the carbon in the African forests, Lewis et al. multiplied Mg of carbon by the range of forested area, 233 to 518 million ha, reported in four surveys. Those low and high figures, multiplied by a fixed average 202 Mg/ha, of course, produced a twofold range of carbon in African tropical forests. Expanding the forest area to the 607 million ha of natural tropical African forest that Grainger (2008) calculated from FRA 2005 would expand the uncertainty. Acknowledging the 174 to 244 Mg/ha confidence interval of Lewis's average 202 Mg/ha would stretch uncertainty even more.

9. Do reports of Chinese forest area and carbon pools agree?

Dong et al. (2003) compared their remotely sensed carbon in Chinese forests with estimates made on the ground by Fang et al. (2001):

> Our estimate for pool size in China (3.68 Gt carbon for the 1995–1999 period) is slightly lower than the estimate by Fang et al. (2001) (4.75 Gt carbon for the 1994–1998 period), and the remote sensing estimate of forest area (142.6 Mha) is slightly greater than the Fang et al. estimate (105.82 for the 1994–1998 period). These differences cause estimates of average pool sizes to differ. Our estimate (25.77 [Mg]/ha for the 1995–1999 period) is smaller than the estimate by Fang et al. … (44.75 [Mg]/ha for the 1994–1998 period). (Dong et al. 2003, 408)

Relative to Fang's values, Dong estimated 35 percent more area, 42 percent less carbon per ha, and a 23 percent smaller stock of carbon in Chinese forests.

10. Might the appearance of accuracy be illusory?

The labor and consequent cost of surveys on the ground and the necessity of standard, uniform definitions and methods for believable global inventories make remote sensing appealing. Satellite imagery has been accepted as a source of ancillary data that can be used with stratified estimation techniques to increase the precision of estimates at low cost (McRoberts et al. 2007). Because area is two-dimensional and a view of the earth from aloft is two-dimensional, testing measurement is rudimentary. A crucial test compares remotely sensed classifications of land with on-the-ground inspections—so-called ground truthing. For example, India tested remotely sensed forest cover with observations on the ground at more than 4,000 randomly selected locations. Satellite and ground classifications agreed at 96 percent of the sites (Forest Survey of India 2008). The accurate remote sensing of Indian forest versus nonforest substantiates the 90 to 92 percent accuracy at 800 points in Costa Rican cited above (Kalacska et al. 2008).

A more ambitious test of remotely sensed values compares *national biomass* or *carbon sums* with alternative estimates. National biomass and carbon have more dimensions than forest area, and they encompass more variety than forest cover at any one place. Nevertheless, remotely sensed regional stocks of carbon sums can be correlated with estimates made from available forest inventories. Correlation between 44 national timber volumes reported by FRA 2005 and the national carbon pools estimated by remote sensing by Dong et al. (2003) accounted for 99 percent of the variation (Figure 3, upper panel).

When the same remotely sensed values were converted to Mg/ha densities by dividing by forest area, however, their correlation with m^3/ha timber densities reported by FRA 2005 explained only 26 percent of their variation (Figure 3, lower panel). The indefinite increase in the estimated carbon as timber densities grow beyond 50 m^3/ha illustrates the saturation of remotely sensed values at high densities (Sanchez-Azofeifa et al. 2009). The correlation in the upper panel of Figure 3 between national values is illusory, reflecting only that large nations tend to have large forests rather than demonstrating that two methods agree.

Not surprisingly, it is difficult to estimate from a remote view how much biomass and carbon a forest contains. This difficulty gives an advantage to other remote sensing technologies, such as LIght Detection And Ranging (lidar), which sense the third dimension, height.

How much carbon tropical forests sequester and release is a mystery that has yet to be solved. They have been poorly measured because they are inaccessible, and although their inaccessibility makes remote sensing attractive, it also leaves the surveyor unable to test whether the remotely sensed value is accurate. Which estimate is correct: the estimate on the ground or from the air? Determining the answer may require comparing remotely sensed values with ground truth among states, provinces, or *Landes* within familiar, well-surveyed nations.

11. How will a global carbon stock be chosen within a 253,000,000,000-Mg range?

We have explored 10 discrepancies among estimates of area, timber, biomass, and carbon in tropical and temperate forests, in large and small nations, and in time and method. Combining uncertain variables, of course, makes global attributes uncertain, too. Because all nations share the same atmosphere and its carbon dioxide, the need to know how much carbon forests have sequestered from our common atmosphere justifies the attempt to calculate the global carbon stock.

Calculation begins with deciding which caches of carbon to include. A table employed by IPCC (Watson et al. 2000) shows the magnitude of caches of carbon that might be encompassed in a global inventory. IPCC cites the source of the table as Wissenschaftlicher Beirat der Bundesregierung Globale UmweltverUnderungen (WBGU 1998), Annex Table 2. WGBU's table corresponds to one compiled still earlier by Dixon et al. (1994). Table 4 of the present paper reproduces the Dixon-WBGU-IPCC quantities in forest vegetation in columns labeled "WBGU."

The global stock of carbon in woody vegetation, reported as 359 petagrams (Pg) by WBGU and copied in Table 4, is only a fraction of the total of 2,477 Pg carbon in vegetation and soil reported by WBGU. The stock of 359 Pg is also only a fraction of the somewhat different 2,300 Pg in vegetation, soil, and detritus, diagramed in Figure 7.3 of the IPCC Fourth Assessment Report (Denman et al. 2007). Forests encompass 1,146 Pg, or about half the global stock in vegetation and soil, according to WBGU. Fully 69 percent of that 1,146 Pg within forest boundaries is in the soil. Finally we come to the trees that a forest inventory would tally: forest vegetation contains only 359 Pg., or 14 percent of the WBGU global total of 2,477 carbon in vegetation plus soil. Nevertheless, 359 Pg is many multiples of the 9 Pg annual global emissions from fuel.

The calculator of carbon stock must decide whether to include tree roots as well as above-ground vegetation, clearly a necessity for harmonizing national inventories (McRoberts et al. 2009). Roots increase nearly linearly as one-fourth of shoot mass (Mokany et al. 2006), making the above- plus below-ground biomass equal to 1.25 of that above ground. The Dixon quantities reproduced by WBGI-IPCC and Table 4 include roots.

Because global stocks will be calculated by multiplying forest area by samples of the density of carbon per hectare, a final question remains: which samples and densities? Table 4, for example, indicates that the area of Russia's high-latitude forests was multiplied by the Mg/ha density of Russian forests to calculate this nation's Pg stock of carbon. The reality of densities can be tested somewhat by comparing Russia's Mg/ha (83) with Canada's (28), Africa's (99), Asia's (132), Europe's (32), and China's (114). Regarding China, Fang et al. (2001) reported that carbon density increased linearly from 15 Mg/ha during 1973–76 to 31 Mg/ha during 1994–98, and more recently, Fang et al. (2007) estimated 41 Mg/ha for China during 1981–2000. These estimates from China are easier to reconcile with the estimates from other countries than is the 114 Mg/ha in Table 4. In the end, the densities adopted by analysts determine the trustworthiness of their calculated stocks for the three broad strata of high-, mid-, and low-latitude forests. And finally, the range of densities across nations and regions calls into question the global sum of 359 Pg in forest vegetation.

Eight years after Dixon et al. made their estimate, one of the coauthors helped prepare another estimate of forest carbon stocks in the northern hemisphere. The Goodale et al. (2002) estimates, which have apparently not been adopted by IPCC, are reproduced in Table 4, under the column heading "Modified." The modified values are less than the WBGU values for all regions except Canada, and considerably less for Russia and China. The modification lowered the estimated global stock in forest vegetation to 302 Pg, or 16 percent less than the WGBU-IPCC value.

The inclusive FAO inventory of global growing stock in 2005 (FAO 2005) found 384 billion m^3. When multiplied by 1 Mg of biomass per 1 m^3 of growing stock, by 1.23 biomass above and below ground per biomass above ground, and by a carbon concentration of 50 percent, the FAO inventory of growing stock corresponds to a global carbon stock of 236 Pg, or 34 percent less than the WGBU-IPCC value.

Such broad strata as in Table 4, of course, are composed in turn of estimates for smaller regions, where Botkin and Simpson (1990) found vegetation densities ranged 10-fold, from 5 to 54 Mg/ha among 12 North American regions of boreal forest. In Russian forests, Shvidenko and Nilsson (2003) found a range of 14 to 54 Mg/ha, and in Finnish forests, Kauppi et al. found 27 Mg/ha. When Botkin and Simpson weighted their North American densities by the 20-fold range of areas of the substrata, they estimated a boreal density of 19 Mg/ha, which is less than five earlier estimates of 27 to 79 in boreal forests. It is also less than the IPCC-WGBU value of 64 Mg/ha for high-latitude forests (Table 4). Botkin and Simpson attributed the discrepancy between their estimate and the higher estimates to other authors' focus on natural forests and forests near universities and cities; Botkin and Simpson said they were estimating carbon density over the whole spread of boreal forest.

Sampling by strata is said to improve estimates. Consider, for example, that seven estimates of the Amazonian carbon stock ranged from 39 to 93 Pg (Houghton et al. 2001). The discrepancy, 54 Pg, in Amazonian rainforests alone looms large, given the 9 Pg annual global emissions from fuel or even the global stock of a few hundred Pg in trees. Unfortunately, sampling by strata in Amazonia will not likely resolve the discrepancy. Four surveys disagreed about where high, medium and low density were, and they found that variation within a forest stand was not clearly less than variation across all Amazonia. Also, and counter to common sense, in seven comparisons the estimated biomass density in deforested areas exceeded the mean density in Amazonian forests as a whole. These results do not engender hope that stratified sampling will improve carbon estimates in Amazonia.

Confronted with a span of 257 to 510 Pg in estimates of the global carbon stock in vegetation, Kauppi (2003) argued that 300 Pg was a reasonable central value. The low, 257, came from FRA 2000 and Fang et al. (2001), and the high, 510, came from two reports published in the 1990s and used by IPCC for calculating the global carbon budget. The 359 Pg for forest vegetation in Table 4 lies within the range confronting Kauppi and is not far from his choice of 300. Converting the 257 to 510 Pg range into a 253,000,000,000 Mg or ton discrepancy reveals its vast scale. And comparing the 253 Pg difference with the annual 9 Pg from fuel emissions, or comparing it with the annual 1.6 Pg emission that IPCC's Figure 7.3 attributes to land-use change like forest clearance, also reveals the scale of the discrepancy. Bravely choosing a value, Kauppi wrote, "Using a resolution of one digit for expressing the estimate, 200 Pg is too low to be realistic, 400 Pg is too high; hence, a stock estimate of 300 Pg is reasonable." Kauppi's words summarize the parlous state of forest inventories.

II. Finding Sources of Uncertainty on the Map of the Forest Identity

What principle can guide an examination of uncertainties of the many and varied attributes of forests? According to their area, forests can harbor biodiversity, beautify the landscape, and bestow solitude. According to their area, forests can also anchor soil, slow erosion, and temper stream flow. Growing stock, which is timber large enough to harvest profitably, furnishes lumber and paper. The forest biomass of leaves, branches, and twigs as well as trunks energizes ecosystems and can fuel economies. Biomass holds carbon dioxide that photosynthesis has subtracted from the sum of greenhouse gases in the atmosphere.

After categorizing these many values into four attributes—area, growing stock volume, biomass, and carbon stock—Kauppi et al. (2006) integrated them in the Forest Identity. The identity relates the attributes to four measurable variables: area, growing stock density, the ratio of biomass to growing stock, and carbon concentration in biomass (Table A, in text).

Table A. Forest Identity Attributes and Variables

Symbol	*Attribute*	*Dimensions*
A	Area	ha
V	Volume = A × D	m^3
M	Biomass = A × D × B	Mg
Q	Carbon = A × D × B × C	Mg
Symbol	*Variable*	*Dimensions*
A	Area	ha
D	Density of growing stock	m^3/ha
B	Allometric biomass ratio	Mg/m^3
C	Carbon concentration	Mg/Mg

Multiplied together, the four variables equal a stock of Q Mg carbon.

Q Mg carbon = *A* ha area × *D* m^3/ha density of growing stock × *B* Mg/m^3 expansion factor × *C* Mg/Mg carbon concentration.

$Q = A \times D \times B \times C$

As the attribute of carbon Q equals the product of all four variables, similarly, the other forest attributes are products of two or three variables.

Although the identity encompasses many valued qualities, still more can be mentioned. For example, the German survey reports naturalness and structure of the forest, species, age, pruning, and damage by game (Bundeswaldinventur 2008; Winter et al. 2008).

During the 1990s, the U.S. Forest Service focus on commercial products in its Forest Inventory and Analysis (FIA) surveys was challenged. In response, the agency began including indices of forest health (Bechtold and Patterson 2005). Guided by the so-called Santiago criteria, the U.S. survey now encompasses indices of biological diversity plus the health and vitality of the ecosystem and its contribution to the global carbon cycle (Smith and Conkling 2004). The Forest Identity and the changes it measures through time encompass many of these indices in four quantitative attributes: area, volume, biomass, and carbon.

For IPCC, the heart of carbon estimation is Equation 2.8b in Chapter 2, Volume 4, IPCC Guidelines (IPCC 2006), which equates the carbon above ground in an ecological zone and climate domain to the same four variables as the Forest Identity. Then, by adding the ratio of all biomass to above-ground biomass, it adds below-ground biomass. The guiding principle of the Forest Identity organizes and maps the following examination of uncertainties.

Area, A

Beginning with area, the Forest Identity identifies the sources of uncertainty in the attributes. The discrepancies of forest area reported above may shake confidence in even this, the first variable of the Forest Identity. Some nations, however, work hard to observe forest area from satellite observations and ground-truth them in statistically sound ways. For example, India inventoried its forest cover, wall to wall, and then compared those classifications of forest and nonforest with observations on the ground (Forest Survey of India 2008).

Annexes of IPCC (2003) describe different surveys of land in six nations and also tabulate eight international data sets. IPCC concentrates its efforts on consistency through time. Acknowledging that standardization is difficult, IPCC Guidelines state,

> Countries will use their own definitions of [land use] categories, which may or may not refer to internationally accepted definitions … Countries should describe and apply definitions consistently for the national land area over time. Countries should describe the methods and definitions used to determine areas of managed and unmanaged lands. All land definitions and classifications should be specified at the national level, described in a transparent manner, and be applied consistently over time. (IPCC 2006, Volume 4, Chapter 3, Section 3.2)

Germany does not survey forests remotely but instead depends on sample plots on a nationwide, 4-by-4-km quadrangle grid (Bundeswaldinventur 2008).[1] The German survey defines a sampling error as follows: "With a probability of 68%, the true value … is the value determined in the sample ± the simple sampling error." That is, the German error is plus or minus 1 standard deviation, as in the U.S. survey. For the forest areas of single *Lander* (states) of about 400 kha, the error ranged from 3 to 4 percent, close to the U.S. sampling error for similar areas of timberland described in the next paragraph.

In 2008 the United States inventoried 46 states. The program includes three sample levels, or phases. Phase 1 is remote sensing for stratification to enhance precision, Phase 2 is based on the original set of approximately one plot per 2.4 kha, and Phase 3 consists of a subsample of Phase 2 plots measured for a broader set of forest ecosystem indicators (approximately one sample location per 38.4 kha). Plots are chosen at random from cells of a grid as illustrated in Figure 4. If the plot is forested, it adds to the estimated forest area; a

[1] The arrangement of sample plots across the nation and the plots themselves are illustrated at http://www.bundeswaldinventur.de/enid/094a41db95eaa06fa5872b3cb029efc2,0/a3.html.

permanent plot is established and periodically revisited. If not forested, it does not add to the forest area but is revisited to observe any regeneration of trees. Sampling proceeds to meet the guideline of 1 standard deviation equal to 3 percent of 405 kha of timberland (U.S. Forest Service 2009; Bechtold and Patterson 2005).

In the end, evidence shows the care and diligence taken by several nations to measure forest area but uncertain outcomes in others. For timberland area in the U.S. FIA surveys, the 67 percent confidence limit, called accuracy, is plus to minus 3 to 5 percent, and the scaling factor for adjustment to various areas is 405 kha. Timberland is the most productive two-thirds of U.S. forestland. For accuracy within 3 percent, therefore, the margin equals 12 kha at 405 kha. The percentage accuracy in ha falls as larger areas are specified and rises for smaller areas. More precisely, accuracy changes in proportion to the square root of (the scaling factor divided by the area). For example, a state with 2,000 kha of timberland would have a margin of 1.3 percent, which equals 26 kha. In another example, a county with only 40 kha would have a 9.5 percent or nearly 4 kha sampling error (Miles et al. 2001; Smith et al. 2002; Bechtold and Patterson 2005).

Forest area depends on what is called a forest as well as on the care expended on measurement, and "'Forest' is a problematic and hybrid category," according to Mather (2005). Although the minimum tree cover that qualifies as *forest* obviously affects the number of hectares, it has not always been agreed upon. Less obvious but also important is the minimum area of tree cover that can be called forest. Although plantations may be intended as sources of lumber, paper, or rubber or as orchards of carbon, they may be excluded from accounting of forest area. Furthermore, bare areas that are normally part of forest area may be called forest if they are expected to revert to trees. Mather (2005, 272) observes,

> More than 650 definitions of forest were assembled in the course of [the global 2000 Forest Resource Assessment by the FAO]. Even within western Europe considerable differences exist. If Spaniards defined forests as the Swedish do, Spanish forests would be 5% larger. Or if they defined forests as the British do, Spanish forests would be 8% smaller.

Applying the U.K. definition would decrease Europe's forest area by 6 percent and applying Norway's would increase it 3 percent (Kohl et al. 2000). Classifying what is and is not forest will plague forest inventories made by any method, ground or remote, and whatever attribute—area, volume, biomass, or carbon—is the goal.

Growing stock density, D

The second variable of the Forest Identity is growing stock density, D m^3/ha. A practical inventory of merchantable timber represented by growing stock was the *raison d'être* of decades of the forestry surveys that are now the heart of forest inventories for all purposes. Although IPCC does not provide tables of growing stock density, it did publish biomass densities (Table 5). Because D equals M/B and, as we shall see in the next paragraphs, B is often near 1, Table A (above, in text) may be read as an illustration of values of D for various climate and ecological types. A two-dimensional view from far above pictures the two-dimensional variable A, but the third dimension, variable D, challenges remote sensing.

The introduction of the Forest Identity emphasized that the *defined* attributes of area and of volume, biomass, and carbon stocks were calculated from four *measured* variables. That is, the identity allows attributes to be deduced from variables. This simplicity must be modified somewhat by revealing that the variables may, in turn, be deduced from measurements of subvariable constituents. Volume can be reasonably estimated from measurements of tree diameter and height. A collection of regressions of Indian eucalyptus volume on diameter and height shows correlation coefficients in the high 90 percent range (National Research Centre for Agroforestry 2008). A forester's handbook (Wenger 1984) shows volume calculated as the product of form factor, diameter squared, and height. In the Forest Identity, the product of the number of trees in diameter classes, their diameters, and heights could replace the variable D m^3/ha growing stock. The underlying correlation of volume with diameter and height will be advantageous for remote sensing by lidar, whose signals per area best give direct measurements of height and somewhat less accurate estimates of basal area (Naessert and Gobakken 2008). We leave the further decomposition of variables into subvariables until the discussion of the expansion factor B Mg/m^3 in the next section, and continue following the outline of the Identity.

A German or American inventory illustrates a growing stock inventory derived from plots distributed across the nation. The U.S. sampling scheme depicted in Figure 4, for example, also produces estimates of growing stock. The description of the Forest Inventory and Analysis database reflects the attention paid to measuring merchantable timber and hence growing stock on the sample plots (Miles et al. 2001). The German inventory places a similar emphasis on growing stock.

The density D of growing stock per area is not easily found in reports but can be calculated from the national reports of area and growing stock by FRA 2005. The 50 nations with the most forest volume in 2005 had densities ranging from 14 m^3/ha in Sudan to 350 in

French Guiana; the 50 densities averaged 77 m^3/ha. Across nations, the FRA 2005 densities had a coefficient of variation (CV) of 57 percent.

The attribute of national growing stock volume V equals $A \times D$ with the combined uncertainty of both attributes. The U.S. Forest Service reported sampling error or accuracy of both area A and growing stock volume V, but not of density D. Here, sampling error and accuracy are equated, and the contribution of bias to accuracy is discussed later. The Forest Service specifies a 5 percent CV for 3.5 million m^3 (1 billion board feet) of growing stock V. That is, 5 percent for 3.5 million m^3 serves for the sampling error of volume V in the Forest Identity (Smith et al. 2002). Germany reports 3 to 6 percent errors in the growing stock of *Landes* and 1.4 percent of the national sum (Bundeswaldinventur 2008). Kauppi (pers. commun.) reports,

> It is fair to say that the forest inventory of Finland is the most advanced national system in terms of accuracy and precision. The country is not very big. and we have a long tradition in this field. Moreover, boreal trees are regular in their shape ("symmetric") and are easy to measure. In Finland an accuracy of plus [or] minus 1–2 per cent has been reached in the estimation of the growing stock within any given subregion of about 500 km^2.

Furthermore, both area A and density D contribute to the uncertainty of V. Thus, uncertainties of less than 10 percent for growing stock in some nations are great achievements and make an assumption of a 5 percent error for national volume V, in later calculations, realistic.

Some nations' inventories, such as those of the United States, are extensive. Although they provide reliable estimates for large areas, their estimates for smaller areas are necessarily less accurate. For a use other than a national estimate, such as getting credit for carbon sequestered by, say, 1 kha forest, another method becomes necessary. Extrapolation of the 3 percent error for 405 kha down to 1 kha leads to an error of fully 60 percent. The U.S. Voluntary Reporting of Greenhouse Gas Program attributes an error of 30 percent to estimates from look-up tables and, using an unflattering word, calls them marginal (U.S. Department of Energy 2006).

Expansion factor, B

The third variable in the Forest Identity, B, Mg of biomass above ground per m^3 of growing stock, converts $A \times D$ m^3 volume into mass. This section examines the variation of B

with density D, values of B around the world, and how the variable B changes with its subvariable constituents of specific gravity and harvest index.

Brown (2002) diagramed the decrease of B as the density of a variety of trees grows, Figure 5. Earlier Brown and Schroeder (1999) expressed the relation by

$$\ln(B) = \beta_0 + \beta_1 \ln(D)$$

For U.S. hardwoods, β_0 equals 1.9 and β_1 equals –0.34, and hence B equals 1.4 when D equals 100 m^3/ha. In Figure 5, tropical hardwoods behave much like temperature ones. For spruce-fir, β_0 is smaller, making B equal to 1.2 at the same density. The B for pine, which responded less to density, is 0.8 or 1.0 at the same density. The standard errors of estimates from the Brown and Schroeder equation are about 0.1, making the coefficient of variation about 10 percent, which is consistent with the range of individual values in Figure 5.

Values of B at D equal 100 m^3/ha calculated by the Brown and Schroeder equation are tabulated in Table 6 (at end) for comparison with other values observed around the world. IPCC Guidelines provide global tables for looking up values of B, which are illustrated in Table 6 by their values at a density D of 51 to 100 m^3/ ha. Table 6 includes Indian values of B (Chhabra et al. 2002).

An unpublished global survey by Fang (2006) found that β_1 ranges from –0.5 in the United States through –0.3 and –0.4 in China, India, and Japan to –0.1 in Europe and Russia. At a reference density of 100 m^3/ha, Fang's B varied from 0.7 to 1.7, a variation and range not inconsistent with the values in Table 6.

Observations of thousands of forest plots show the frequency distributions of B and its change β_1 with density. Coefficients fitted to about 100 thousand forest inventory plots across the United States by Smith et al. (2003) relate biomass density M/A to growing stock density D. Seventy-three of their forest types and regions allowed calculations of M/A and D. Because B equals $(M/A)/D$, Smith et al. values also allowed calculation of B. The frequency distributions of B at a density of 100 m^3/ha and of its change β_1 with density in Figure 6 illustrate the variation of B in the Forest Identity. These values for B average 1.1 and range from 0.6 to 1.8, not unlike other B values already cited. Because B in Figure 6 encompasses many types of trees, it is not surprising that the coefficient of variation of 21 percent exceeds the approximate 10 percent of values from Brown and Schroeder's equation for one type of tree.

The subject now shifts to how the variable B changes with its subvariable constituents of specific gravity and harvest index. Specific gravity and harvest index have already appeared in Table 6.

The Mg and m^3 dimensions of the allometric variable B reflect its dual functions. Decomposing B into its dual subvariables shows the roots of B's behavior. One part, the specific gravity ρ Mg/m^3 of wood, translates growing stock volume into growing stock mass. Because allometry is defined as the relative growth of a part in relation to an entire organism, specific gravity is not truly an allometric parameter. On the other hand, the relative growth f of the mass that is valuable timber Mg in relation to the entire biomass Mg is truly the allometric part of B.

The allometric ratio of grain to biomass above ground is called the harvest index in agronomy. Its rise from about a third to a half explains much of the grain yield increase during the 20th century. For a forest, the fraction f of merchantable growing stock mass per total biomass above ground is analogous to the harvest index. Thus, $B = \rho / f$. At the improbable limit that all biomass becomes wood, f becomes 1, making B equal to the specific gravity ρ. Because the specific gravity of timber is often near one-half (Birdsey 1992), the lower limit of B is about one-half. The apparent lower limit of B equal to 1 in Figure 5 naturally exceeds the theoretical limit of one-half that would be achieved if all the biomass were in the dry matter of the growing stock and the trees had no leaves, twigs, and so forth. If ρ equals 0.5, the apparent limit of B equal 1 implies a lower limit of f equal to 0.5, meaning that in dense forests, the merchantable timber holds half the Mg of the biomass. The harvest index of half for timber in dense forests thus compares favourably with the harvest index of grain crops. In Table 6, illustrative specific gravities are shown with B and the harvest indices of 0.3 to 0.9 that they imply. This range brackets the harvest indices near 0.5 implied by Figure 5.

The introduction of the harvest index f helps visualize a changing B as a forest grows denser in idealized Figure 7 and also visualize the relation of B to real density D in Figure 5, above. Imagine an idealized forest where, beginning at time 0, small trees grow along the straight green line in Figure 7. At first, during each time unit, they grow 1 unit of M/A biomass per area. Initially, no trees are large enough for timber, f is 0 and B is undefined. Later, after time 1, trees grow large enough to be harvested, causing density D to grow along the straight blue line at 1 unit per unit time. The linear growths of M/A Mg/ha and D m^3/ha raise the fraction f in timber along the curvilinear red line. The hypothetical B biomass per volume falls curvilinearly along the black line. Measured B frequently declines from 5 to 1 Mg of biomass per m^3 of timber, as in Figure 5. Because f equals ρ/B and ρ equals 0.5, allometric ratios from 5 to 1 in Figure 5 represent volume holding a 10th to half of the above-ground biomass. A real rather than idealized forest would grow along S-shaped curves rather than the straight lines, and changing specific gravity would affect the curves. Nevertheless, the simple straight lines for density D of growing stock and M/A of biomass demonstrate how the curves of B and f get their shapes.

The subvariables ρ and $1/f$ accompanying the variable B in Table 6 can now be understood. For the United States, Birdsey (1992) compiled specific gravity ρ across 23 regions and found averages of 0.43 for softwood and 0.52 for hardwood with a 17 percent coefficient of variation. The ρ of 93 U.S. woods averaged 0.52 with a 23 percent CV, and the ρ of 56 imported woods of diverse species averaged 0.61 with a 29 percent CV. Across eight regions, Birdsey (1992) estimated harvest indices f of 0.6 for softwoods and 0.5 for hardwoods. The reciprocals $1/f$ averaged 1.8 and 2.0 with 14 percent CV.

In Table 6, the allometric harvest index f was calculated from assumed specific gravity and B. The assumed specific gravity for softwoods was of course lower than for hardwoods, but the harvest indexes for different types overlapped. The range from smallest to largest B was fourfold, but the subvariables specific gravity and harvest index still ranged two- to threefold. The biomass ratio B and its components ρ and $1/f$ add uncertainty to forest inventories.

Carbon concentration, C

IPCC presents a default carbon concentration of 0.47 and a range from 0.43 to 0.55 (IPCC 2006, Table 4.3). For the United States, Birdsey (1992) reported carbon concentrations from 0.51 to 0.53 in softwoods and 0.50 to 0.51 in hardwoods, with a coefficient of variation of 1.1 percent. Although carbon concentration adds little uncertainty, the other variables of the Forest Identity allow ample opportunity for uncertainty in inventories of forest attributes.

III. Integrating the Uncertainties of Variables into Uncertainties of Attributes

The uncertainties of the variables A, D, B, and C of the Forest Identity can now be assembled into the uncertainty of their products, the attributes of growing stock volume $V = A \times D$, of biomass $M = A \times D \times B$ and of carbon stock $Q = A \times D \times B \times C$. Weighing the relative addition to the uncertainty of an attribute by each variable can guide research to the less uncertain attributes.

Every estimate has three parts: the estimate itself, a margin of error, and the probability that actuality lies within the margin. As illustrated above for area and density, uncertainty or margins of error are often written as the 67 percent or 95 percent probability of actuality falling within 1 or 2 standard deviations of the estimate. For example, see 67 percent probability and 1 standard deviation at Smith et al. (2002), or see 95 percent probability and 2 standard deviations at Brown et al. (2007).

If the final estimate is a *sum* of uncorrelated observations estimates, then the variance of the final estimate is the sum of their variances. For example, Brown et al. (2007) show an example of the density of above-ground carbon in a forest; it is the sum of carbon in living trees, dead wood, and litter. In their example, the variance and margin of error of the sum of the three sources lies close to that of the single component of living trees, the most variable or most uncertain component. Their lesson is that *refining the most uncertain component affects the certainty of the sum greatly*.

In the Forest Identity, however, the attributes are *multiplied*, not summed. For example, the attribute of growing stock volume *V* is calculated by multiplying variables of area *A* by density *D*. The variances of the component variables must be weighted by their means before they are combined into the variance of the attribute. Thus the square of the coefficient of variation (CV^2) of the product, an attribute, equals the sum of the squares of the component CV^2 of variables (Goodman 1960; Freese 1962). For multiplied components, Brown et al. (2007) explain with an example of the carbon in a forest estimated as area multiplied by density. They demonstrate that the uncertainty of the least certain variable, density, overwhelms the certainty of the area. They warn, "If uncertainty is not equally low for the two sources of the ultimate deforestation and degradation emissions, then the investment in the [more certain] half is money poorly spent."

Because the attributes of biomass *M* and carbon stock *Q* are products of variables, seemingly small errors in the absolutely small variables *B* and *C* leverage large errors in attributes. For example, an error in the area of U.S. forest (303 million ha) three times as large as the state of Indiana (9.43 million ha) would affect U.S. biomass or carbon stock slightly less than an error of only 0.1 in a biomass ratio *B* of 1.0. This same error would affect carbon stock slightly less than 0.05 errors in specific gravity, harvest ratio or carbon concentration of 0.5. That is, because area *A* and the ratio *B* are multiplied to calculate biomass M and carbon stock Q, a 10 percent error in little *B* equals a 10 percent error in big *A;* and the area of Indiana is less than 10 percent of U.S. forest area.

Laboring to improve the certainty of the least uncertain components wastes effort, and the variables that are small in absolute values can have great leverage.

The preceding paragraphs about the variability of products must be qualified if the component variables are correlated. A negative correlation, like that between *B* and *D* evident in

Figure 5, narrows the margin of error of their product, the biomass density $B \times D$ Mg/ha. A negative correlation between ρ and 1/*f* would narrow the margin of error of their product, the ratio *B*.

The introduction of specific gravity and harvest index permits a deeper analysis of the effect of *B* on uncertainties. The Forest Identity can be rewritten to accommodate ρ and 1/*f* . It can also be rewritten to accommodate the specification of errors for *V* (as the German and U.S. inventories do) rather than the errors of *D.*

$Q = A \times D \times B \times C$ becomes

$Q = V \times \rho \times 1/f \times C$

The U.S. Forest Service specifies a 5 percent coefficient of variation for 1 billion board feet of growing stock; that is, 5 percent for 28 million m^3 serves as the sampling error of *V* (Smith et al. 2002). German errors for *V* are consistent with 5 percent. The accuracy of measurement in some nations cannot compensate for the discrepancies in area and volume estimates elsewhere, as described in the first section of this paper. Nevertheless, a 5 percent error for volume is not unattainable or unreasonable—recall the 1 to 2 percent accuracy of the Finnish inventory—and is assumed for combining uncertainties.

Let the coefficient of variation of specific gravity ρ be 17 percent, of harvest indices 1/*f* be 14 percent, and of carbon concentration be 1.1 percent as calculated above from the Birdsey (1992) report.

Ignoring any correlation among *V* and the three variables, their uncertainties (Table 7, line 1) were combined according to the expression for the Forest Identity and the rule that the CV squared of their product equals the sum of the CV squared of its constituents. Pass over the small absolute values of the CV^2 in the first line of the tabulation and concentrate on the second line, the percentage contributions to the uncertainty of the product *Q*. The outcome clearly highlights the surprising importance of uncertain ρ and 1/*f*.

Does the separate addition of CV for ρ and 1/*f* inflate the impact of $B = \rho/f$? Combining the CV in Table 7 for wide ranges of trees produces a CV of 22 percent for *B*, close to the 21 percent CV calculated for *B* estimated directly for the wide range depicted in Figure 6. The lesson that uncertain ρ and 1/*f* have surprising importance remains.

A logical objection can be raised to comparing the uncertainty of volume *V* in actual national inventories with the CV of variables in tables spanning forest types and regions. The small variation in carbon concentration *C* dispels any objection to its comparison with that variation of the attribute *V*. The agreement between the CV of *B* and the combined CV of ρ and 1/*f* answers objections to substituting their combined uncertainty. Objection may still be raised against comparing the variation of *B* from Smith et al. (2003) and Figure 6 with the variation of *V* in actual inventories. That objection can be partially met by lowering the CV of *B* to the 0.1 standard error of *B* around Brown and Schroeder's (1999) regression of *B* on *D*. The first important consequence is that the calculated CV shrinks from 22 to 14 percent, demonstrating the effectiveness of lowering the uncertainty of the most uncertain component. The second consequence is that the CV of *A* and *B* now each contribute half to the uncertainty of *Q*. The lesson still remains: seemingly small variations in *B*, like small variations of its components ρ and 1/*f*, have a large impact on estimates of biomass or carbon.

IV. A Dangerous Omission from Thoughts about Accuracy

The previous section about integrating uncertainties included three parts: the estimate itself, a margin of error, and the probability that actuality lies within the margin. An omitted and dangerous fourth part concerns whether the laboriously acquired precision of estimate, margin of error, and probability represents the whole intended population or only part. The fourth part of accuracy is bias, described as follows in a classic work on forest measurement:

> Among statisticians accuracy refers to the success of estimating the true value of a quantity; precision refers to the clustering of sample values about their own average, which, if biased, cannot be the true value. Accuracy, or closeness to the true value, may be absent because of bias, lacking precision, or both. A target shooter who puts all of his shots in a quarter-inch circle in the [bullseye] might be considered accurate; his friend who puts all of his shots in a quarter-inch circle [far from the bullseye] would be considered equally precise but nowhere near as accurate. (Freese 1962, 4)

An exercise with Shvidenko and Nilsson's (2003) estimates of above- and below-ground vegetation in four zones of Russian forest dramatizes the consequences of bias. The carbon densities were 14 Mg/ha in tundra; 23 Mg/ha in forest tundra and northern and sparse taiga; and 45 and 54 Mg/ha in middle and southern taiga. In the extreme bias of all samples being drawn from one of the four zones, the estimate of carbon in boreal Russian forests would have been 10, 17, 33, or 39 Pg instead of the 31 Pg calculated by adding products of zonal areas and densities.

An inventory of North American boreal forests provides a real example of avoided bias. Botkin and Simpson (1990) divided these forests into 12 strata and appraised the carbon in vegetation in random samples within each stratum. This stratified random sampling produced an average carbon density of 19 Mg/ha, well outside the range of 27 to 79 Mg/ha of five earlier estimates. The carbon stock of 22 Pg estimated from stratified random sampling was smaller than the 30 to 79 Pg of the five earlier estimates. Botkin and Simpson rationalize the discrepancy this way:

> We believe that our estimates are much lower than previous ones for reasons related to methodology and disturbance. Some of the earliest estimates were made when almost no quantitative data were available and the data or the estimates were largely speculative. Other estimates of boreal forest above-ground biomass were obtained by extrapolating data of small restricted studies to large areas. This was a biased approach which led to overestimation, first because ecologists and foresters who made those measurements did not have total areal estimate as their goal. They usually sought to study significant disturbance or bare ground. As a result, studies of biomass were typically conducted in undisturbed mature (i.e., high biomass) forests which were not representative, because the boreal forest is a large and heterogeneous mosaic, composed of a patchwork of vegetation stands in various successional stages ... Second, most early studies were conducted near universities and cities which generally lie in the more southern regions of the boreal forest where biomass density is greatest ... Extrapolation from these studies invariably leads to overestimates of the total biomass. (168)

The 64 Mg/ha density used by IPCC for boreal forests (see Table 4) certainly exceeds the 19 Mg/ha estimated by stratified random sampling of North American boreal forests.

V. Conclusion

The 11 discrepancies described at the beginning of this paper portray the parlous state of global forest inventories. Deciding how large a group of trees and what kind of vegetation merits the label *forest* adds to the uncertainties of either sampling forests on the ground or viewing them remotely. The discrepancies illustrate the dangers of asserting that forests are facing disaster or, on the contrary, that they are improving. The discrepancies make suspect any promises that a nation's forests will sequester a specified tonnage of atmospheric carbon dioxide.

Nevertheless, if one can tolerate an uncertainty of 3 to 10 percent of, say, 400 kha or 4 million m^3 of growing stock volume specified and written in a contract, the methods employed in several developed nations can fill the requirement. The cost of achieving 3 to 10 percent accuracy is necessarily the next question, and the forest surveys of the United States provide an example. In 2002, one-third of all U.S. land, or 300,000 kha, was forested, and 200,000 kha consisted of the productive sort called timberland (Smith et al. 2002). In 2007, federal expenditures plus money from cooperators for the forest survey totaled $72 million, including the cost of employing 562 person-years (U.S. Forest Service 2009). Allocated to forest area, the $72 million cost equals $24 per km^2, or 24 cents per ha. Table 8 shows the costs of surveys calculated at $24 per km^2 and the forest areas of the 12 nations with largest forest areas in 2005 plus two nations with thorough surveys. Though only an order-of-magnitude calculation, it shows the scale of expense. The calculated costs range from nearly $200 million for Russia and more than $100 million for Brazil to $14 million for Bolivia and $10 million for Zambia. At the same rate, Finland and Germany would spend $2 million to $5 million. Evaluating these costs versus accuracies that are good enough will guide nations and others in choosing survey methods. The Design and Implementation of Effective Measurement and Monitoring project, mentioned at the beginning of this paper, will report on requisite accuracy and methods, especially remote sensing, in other RFF publications.

A glance at an aerial photograph heightens the attraction of measuring the two-dimensional attribute of forest area *A* from aloft, and tests like those in India provide reassurance that forest cover can be remotely estimated. Turning back to the Forest Identity, however, we have the sobering realization that forest attributes have at least three more dimensions, *D, B,* and *C*.

Well-run on-the-ground surveys of national growing stock volume *V* have accuracy closer than 10 percent. These accurate surveys of *V* divided by equally accurate measurements of area *A* provide density *D* with reassuring accuracy.

Although perfect accuracy might seem the goal, it is not—at least not in the real world of affairs. Rather, the cost of improving accuracy makes *good enough* the goal. If the costs of surveying, monitoring, and verification exceed the consequent benefit or profit, regulation will fail and transactions abort in the long run. In an examination of so-called transaction cost for sequestering carbon in forests, monitoring and verification were the most expensive category in 6 of 11 examples (Antinori and Sathaye 2007). Thus the discrepancies and uncertainties in forest surveys must next be evaluated against standards of good enough for, say, scientific debates, timber sales, or carbon credits. Then economical methods for meeting that standards must be established.

Multiplying reliable V by insecure ratios of growing stock to biomass, however, introduces uncertainty. A seemingly small uncertainty of, say, 0.1 to 0.2 in the ratio B can exert as great a leverage on the product of the multiplication as an uncertainty of thousands of hectares. The product of the multiplication may be either biomass $M = A \times D \times B$ or carbon $Q = A \times D \times B \times C$. B itself is a ratio of two components: specific gravity (the mass of wood per volume) to harvest index (the ratio of growing stock mass to biomass). Tabulated values of either of the two components of B show their capacity to inject uncertainty in B. Unless other influences narrow the wide range of tabulated specific gravities of wood and harvest indices, the consequent uncertainty may invalidate biomass and carbon estimates, even after accurate area and growing stock volume measurements have been achieved—at great trouble and expense.

Ending a discussion of accuracy with only the precision of estimates evident in their variances, standard errors, and coefficients of variation would be dangerous. Common sense includes a lack of bias along with precision in the desired quality of *accuracy*. One might, for example, multiply precise measurements of the Mg/ha of carbon in sampled hectares by the hectares of Amazonian rainforest. Before trusting the product as the stock of carbon in the Amazonian rainforest, however, one must ask whether every hectare in this forest had an equal chance to be included in the precise measurement of Mg/ha. If not, the multiplication is misleading.

After evidence of arresting discrepancies and sources of uncertainty, the standards of "good enough" accuracy must be established and the remedies for a shaky foundation of forest inventories must be prescribed.

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Tables and Figures

Table 1. Deforestation in Brazil during 2000 to 2005, Reported by Three Organizations

		All land kha	Forest 2000 kha	Loss 2000/2005 kha/5 yr	Loss % of 2000 %/5 yr
FRA2005	Nation	851,488	492,213	15,515	3.2%
Hansen	Nation	851,488	492,213	13,000	2.6%
PRODES	Nine states of Amazonia	513,974	335,349	11,070	3.3%
Hansen	Humid tropical	530,135	361,160	13,002	3.6%

Source: Source: FRA2005, Earth Observation Coordination (2008), Hansen et al. (2008). The author thanks Dalton Valeriano, Luis Murano and Ruth DeFries for help but accepts responsibility for the table.

Table 2. A Comparison of Directions of Changing Forest Area Assayed by Three Alternative Methods: FRA assessment reports published by FAO, reports submitted to FAO for preparation of assessments, and actual surveys by nations recognized as valid by them in their reports for assessments.

Nation	Reports by FRA	Reports to FRA	Survey by Nation
Costa Rica	Possible Reversal	Rise	Uncertain
Guatemala	Decline	Reversal	Uncertain
Honduras	Decline	Decline	Decline
Mexico	Decline	Uncertain	Uncertain
Madagascar	Decline	Reversal	Decline
India	Possible Reversal	Rise	Reversal
Nepal	Decline	Reversal	Decline
Vietnam	Reversal	Decline	Reversal

Source: Grainger (2009).

Table 3. Comparisons of Six IPCC Tier 1 and IPCC Good Practice Measurements of Carbon Density Units of t tons C/ha equal Mg carbon/ha.

Location	IPCC Forest type by region	Tier 1 estimate Mg/ha	Plot measurement Mg/ha	Tier 1 as % of plot measurement %
Brazil	Tropical rainforest	150	218	-31
Indonesia	Tropical rainforest	175	212	-17
Rep Congo	Tropical rainforest	155	277	-44
Rep Guinea	Tropical rainforest	155	209	-26
Madagascar	Tropical rainforest	155	148	+5
Mexico	Temperate mountain	65	49	+33

Note: Tier 1 estimated by forest type and continent.
Source: Brown et al. (2007) box 4.2.

Table 4. Forest areas and forest carbon stocks and densities. The areas and carbon in column *WBGU* were reported successively by Dixon, WBGU and IPCC. Columns *Modified* show values reported by Goodale et al. for three high latitude nations and for the U. S. A., Europe, and China. A few choices were made in presenting columns *WBGU*. Dixon's carbon stock for Asia was chosen to correspond with the mid latitude sum, and the 114 Mg/ha density was retained as published rather than changed to the ratio of carbon stock to area. Sources: Dixon et al. (1994), Goodale et al. (2002), Watson et al. (2000), WBGU (1998).

	M ha	Carbon stocks Pg		Carbon density Mg/ha	
		WBGU	Modified	WGBU	Modified
Forest High Latitude					
Russia	884	74	*33.7*	83	38
Canada	436	12	*12.9*	28	30
Alaska	52	2	*0.6*	39	12
Total/Average	1,372	88	*47.2*	64	34
Forest Mid Latitude					
USA	241	15	*12.7*	62	53
Europe	283	9	*7.7*	32	27
China	118	17	*4.6*	114	39
Australia	396	18	18	45	45
Total/Average	1,038	59	*43*	57	41
Forest Low Latitude					
Asia	310	41	41	132	132
Africa	527	52	52	99	99
South America	918	119	119	130	130
Total/Average	1,755	212	212	121	121
Global					
Total/Average	4,165	359	302.2	86	73

Table 5. Default densities of biomass. Biomass density equals $D \cdot B$. Thus if B is near 1, the tabulated values illustrate the range of D m^3/ha as well as biomass density $D \cdot B$. Source: IPCC (2006), Table 4.

Climate domain	Ecological zone	Above-ground biomass in natural forests (tonnes d.m. ha^{-1})
Tropical	Tropical rain forest	300
	Tropical moist deciduous forest	180
	Tropical dry forest	130
	Tropical shrubland	70
	Tropical mountain systems	140
Sub-tropical	Subtropical humid forest	220
	Subtropical dry forest	130
	Subtropical steppe	70
	Subtropical mountain systems	140
Temperate	Temperate oceanic forest	180
	Temperate continental forest	120
	Temperate mountain systems	100
Boreal	Boreal coniferous forest	50
	Boreal tundra woodland	15
	Boreal mountain systems	30

Table 6. Examples of B Mg/m^3 at density D near 100 m^3/ha and illustrative values of specific gravity Mg/m^3. Pairs of B and specific gravity imply the tabulated harvest indices Mg/Mg. Source of B: *a*-Brown (2002), *b*-IPCC (2006) *c*-Chhabra, Palria and Dadhwal (2002). Source of specific gravity: Birdsey (1992).

		B	Specific gravity	Harvest index	
Locale	Type	Mg/m3	Mg/m3	Mg/Mg	Source
Temperate	Pine	0.6 to 1.0	0.4	0.4 to 0.7	b
Temperate	Pine	0.8 to 1.1	0.4	0.4 to 0.5	a
Humid tropical	Pine	0.6 to 1.2	0.4	0.5 to 0,7	b
Indian	Pine	1.7	0.4	0.4	c
Temperate	Spruce	1.1 to 1.2	0.4	0.3 to 0.4	a
Indian	Spruce Fir	1.2	0.4	0.3	c
Temperate	Hardwoods	0.7 to 1.9	0.6	0.3 to 0.9	b
Temperate	Hardwoods	1.1 to 1.5	0.6	0.4 to 0.5	a
Indian	Hardwoods	1.4	0.6	0.4	c
Tropical	Hardwoods	1.3 to 2.0	0.7	0.4 to 0.5	a
Humid tropical	Natural forest	1.0 to 2.5	0.7	0.3 to 0.7	b

Table 7. The great effect on the coefficient of variation CV of the attribute of carbon stock Q by specific gravity r, reciprocal harvest index $1/f$ and B, and the lesser effect of volume V and carbon concentration C. In the upper table, calculation proceeded from the sub variables r and $1/f$.; in the lower table it proceeded instead from the variable B. The CV^2 of Q in the final column is the sum of the CV^2 in the preceding columns. The rows labelled (% of CV^2(Q), relate the contribution of factors to the CV^2 of the carbon stock Q.

CV of Q calculated from ρ and 1/*f*					
	V	ρ	1/f	C	Q
CV	5%	17%	14%	1.1%	22%
% of CV2(Q	5%	57%	37%	<1%	100%
CV of Q calculated from *B*					
	V	B		C	Q
CV	5%	10%		1%	11%
% of CV2(Q	20%	80%		<1%	100%

Table 8. The national costs of surveying forests calculated at \$23.75 per km^2. Twelve nations with the largest forest area in 2005 according to FRA2005. Two European nations with thorough surveys. The cost per km2 is the ratio of the USA survey cost divided by its forest area. Source:

	Forest area	Cost
	kha	Million $/yr
Russia	808,790	192
Brazil	477,698	113
Canada	310,134	74
USA	303,089	72
China	197,290	47
DR Congo	133,610	32
Indonesia	88,495	21
India	67,701	16
Sudan	67,546	16
Angola	59,104	14
Bolivia	58,740	14
Zambia	42,452	10
Finland	22,500	5
Germany	11,076	3

Figure 1. Forest cover from Mexico to Panama classified by GLC2000 and by MODIS (lower panels) and the disagreement between them. The red circle identifies a hot spot of disagreement in Guatemala and El Salvador. Source: Geo-WIKI 2009.

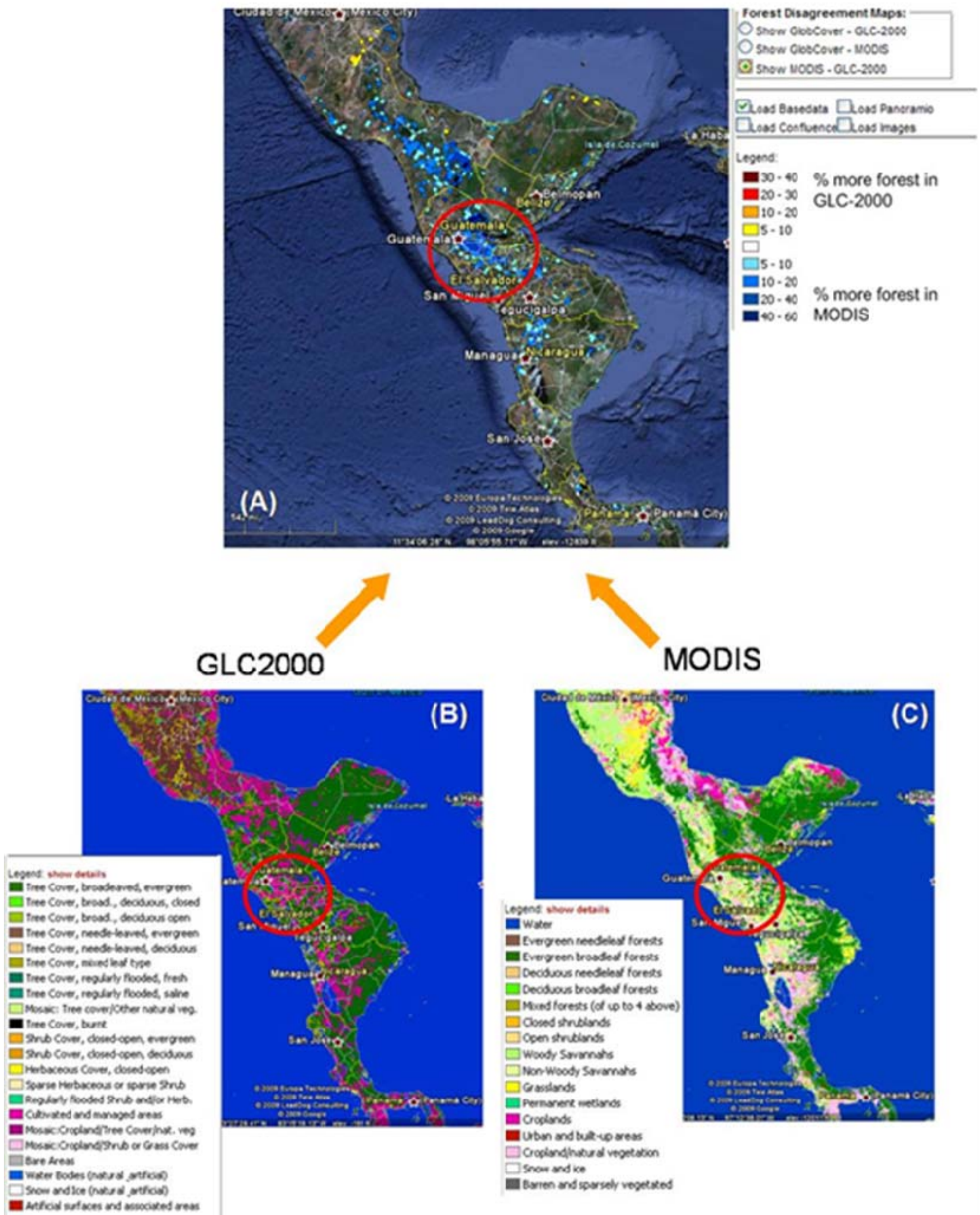


Figure. 2. A synoptic chart of changing forest area *a* %/yr. versus forest density *d* %/yr. The chart depicts changes from 1990 to 2005 in the 50 nations with the largest forests in 2005. A copy of Kauppi et al. (2006), figure 5.

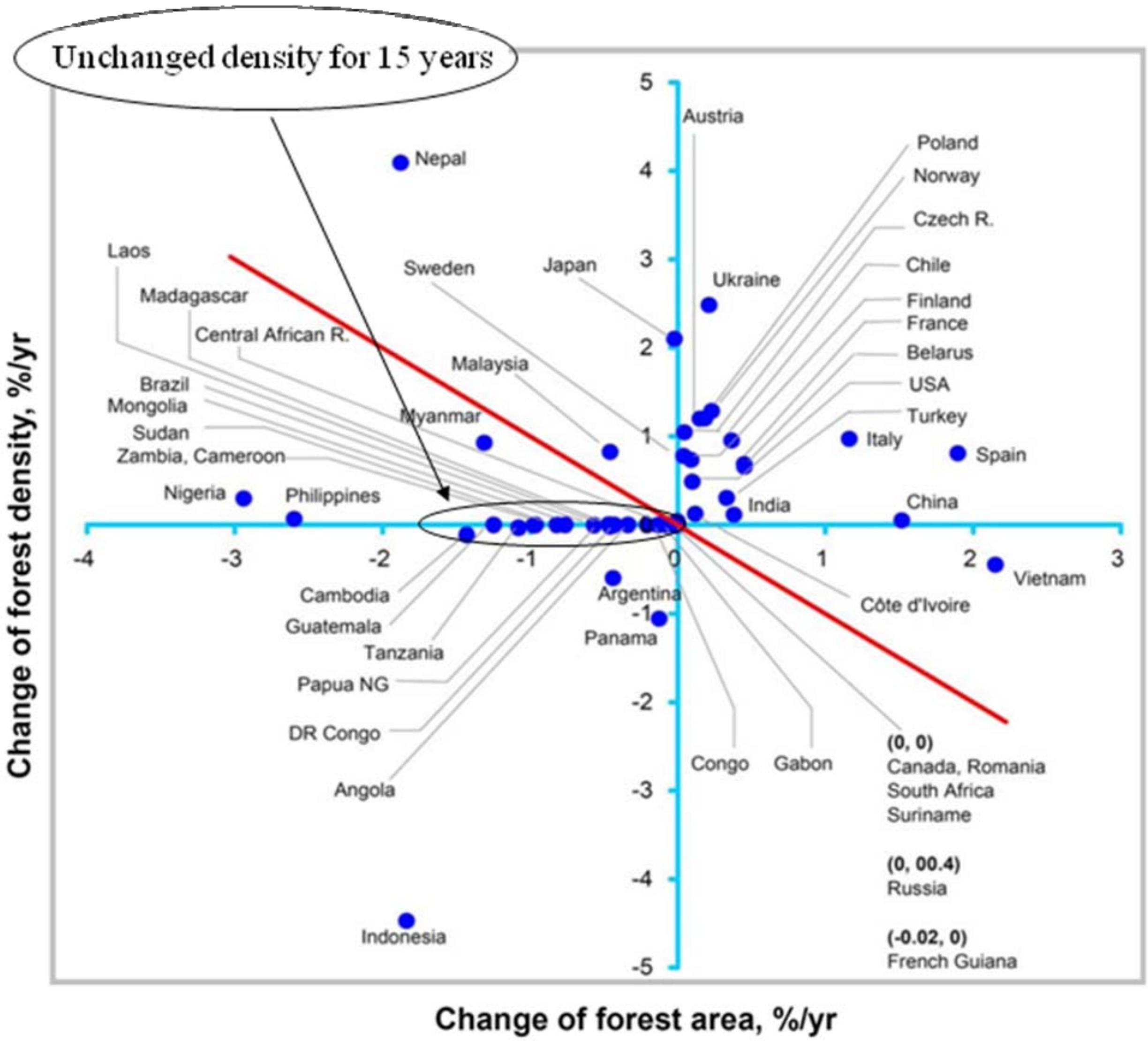
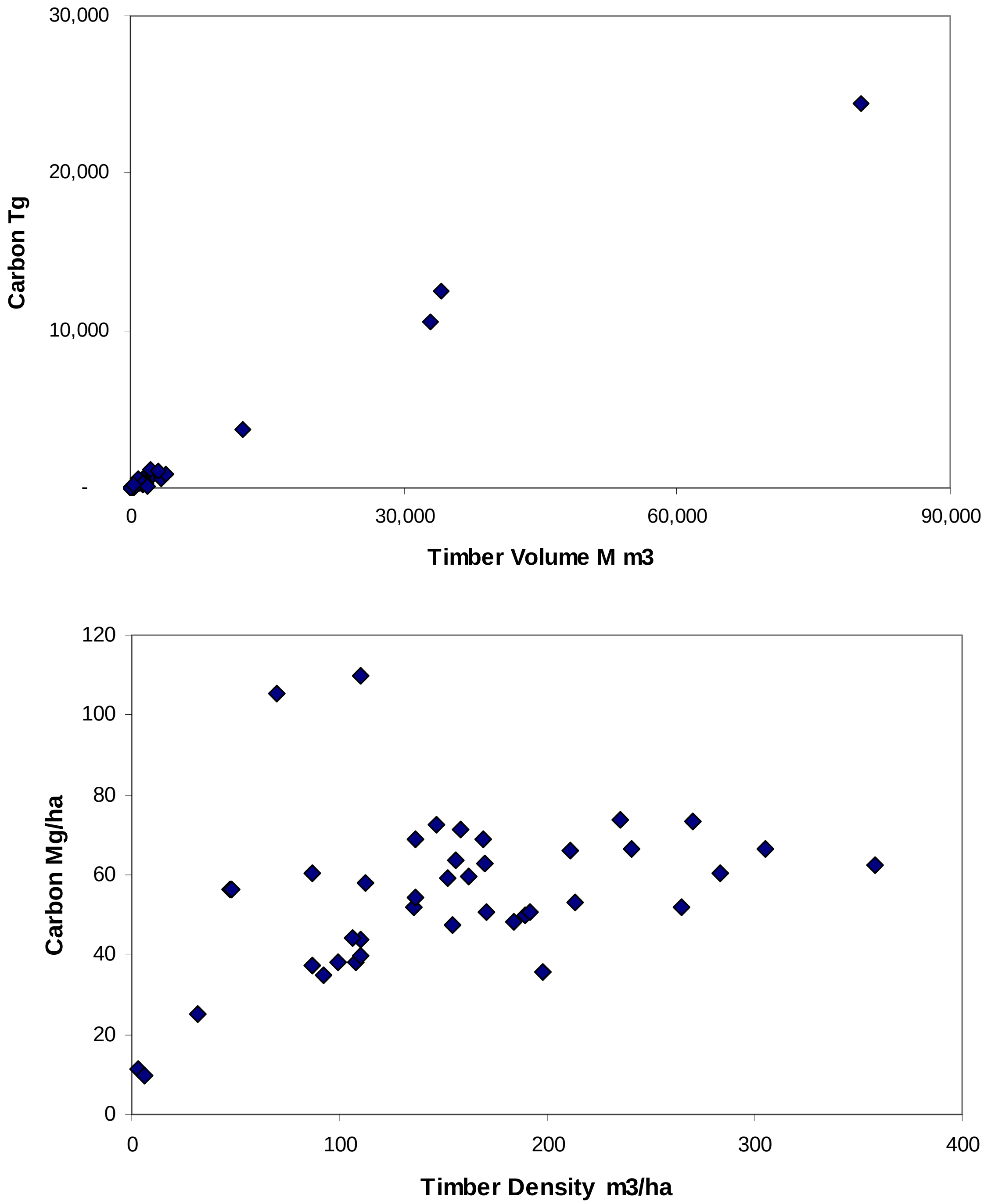
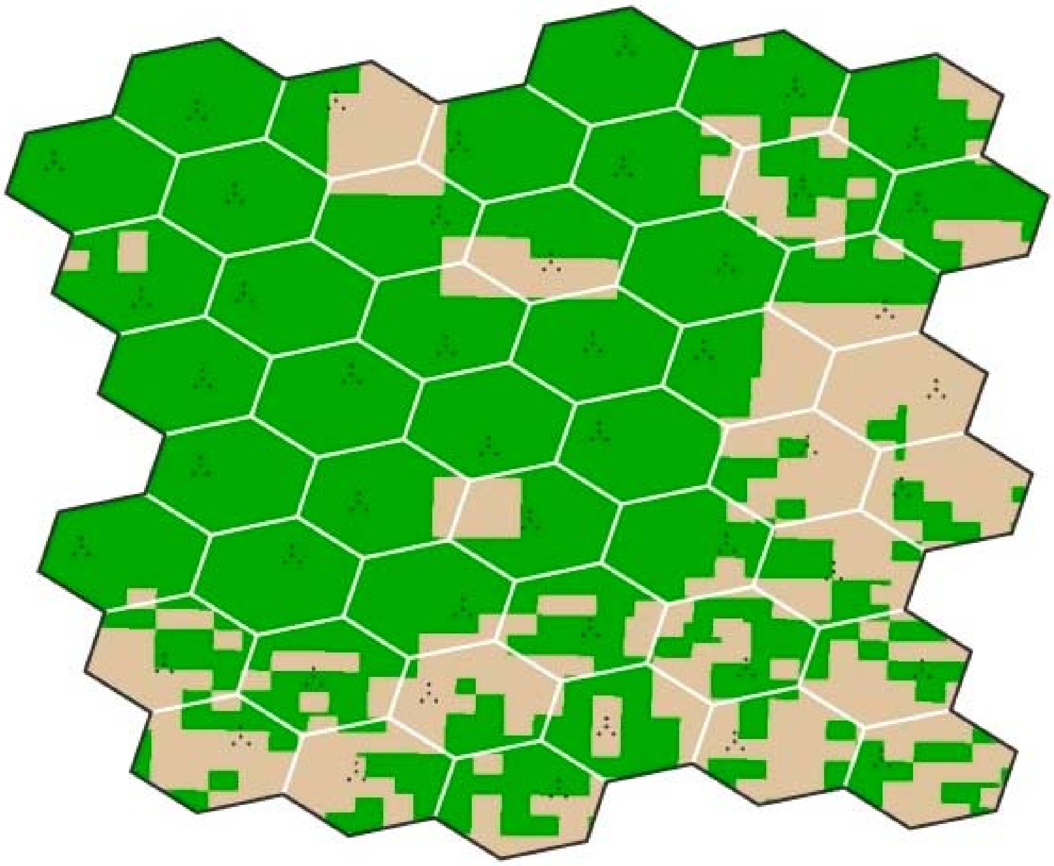


Figure 3. The correlation between 44 national pools of carbon sensed remotely and timber volumes calculated from FRA2005 (upper panel). The lack of carbon density per ha and timber volume per ha (lower panel).



Source: Dong et al. (2003 Table 3), FRA2005

Figure 4. An illustration of the U.S. sampling frame for forest survey superimposed onto a landscape.



Note: The patchwork of green and brown under the hexagonal grid represents forest and non-forest areas. Each hexagon contains one ground plot symbolized by the cluster of four points located within each hexagon. The ground plots are not drawn to scale.
Source: Bechtold and Patterson (2005).

Figure 5. Observations of the ratio B Mg/m^3 of biomass as a function of growing stock density D m^3/ha. Source: Brown (2002).

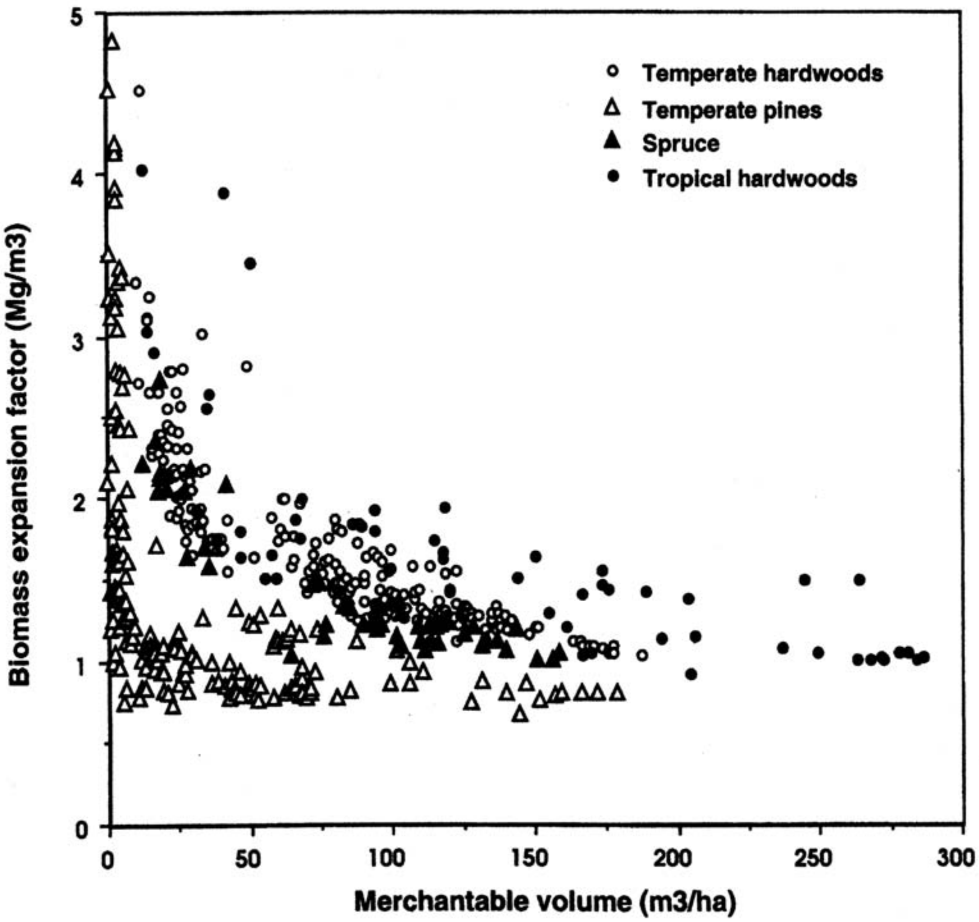
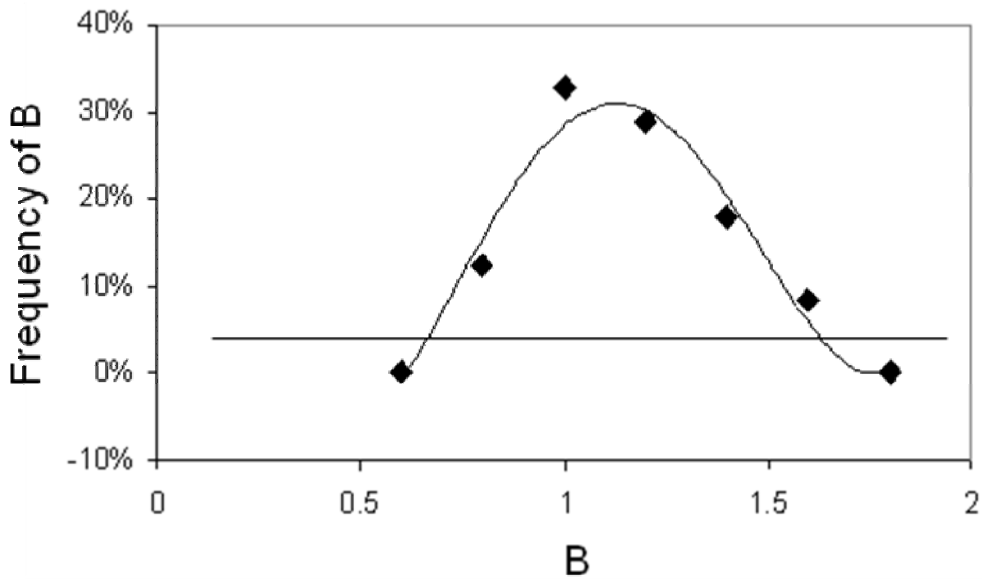


Figure 6. The frequency distributions of B at a density D of 100 m^3/ha (upper panel) and of its change β_1 with density (lower panel) in plots across the USA. Source: Smith, Heath and Jenkins (2003).



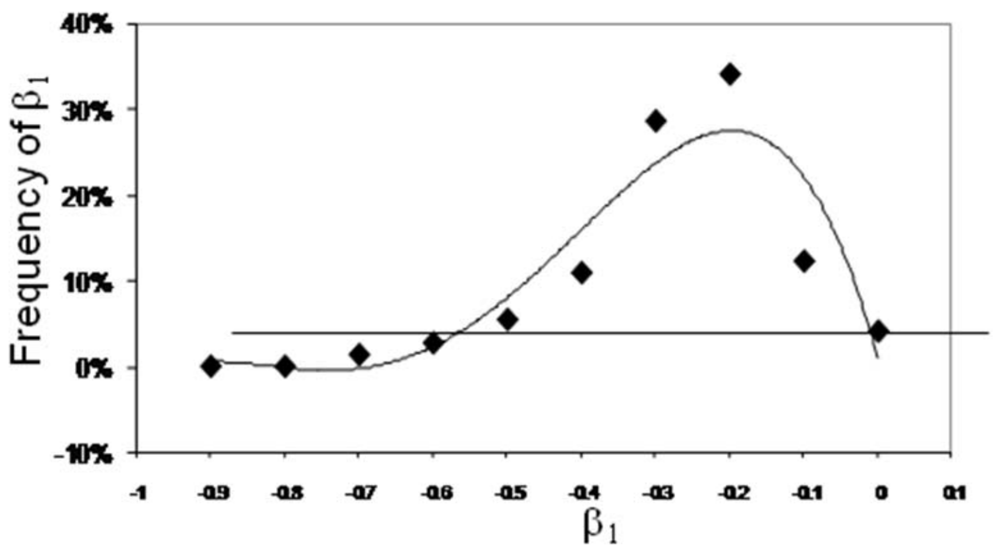


Figure 7. An idealized forest where the biomass per area M/A grows 1 Mg/ha per time from time 0, whereas timber density D grows at 2 m3/ha, but after time 1. The ratio f represents (Mg of merchantable timber per Mg of biomass), and over time it approaches 1. As the trees grow, B (tons of biomass per m3 of volume) approaches the specific density of wood, near 0.5. Source: Waggoner and Ausubel (2007)