Multi Acoustic Prediction Program (MAPP^{tm}) Recent Results

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1. Introduction

The Meyer Sound Multi Acoustic Prediction Program (MAPP^{tm}) is an ongoing research program to develop and distribute sophisticated software for accurately modeling the acoustic interaction of loudspeakers.

Currently, we use a far-field model of acoustic interaction, based on the empirically measured far–field polar patterns of loudspeakers. Using a rotation device in an anechoic chamber, we position the loudspeakers every degree both horizontally and vertically, for a total of $360 * 180 = 64,800$ measurement locations. We use a multiple timescale FFT based measurement system, which gives us at least $\frac{1}{24}$ octave resolution in the audio spectrum 20Hz $\rightarrow$ 20kHz. The resulting data sets are large, approximately $1.5 * 10^9$ bytes, (1.5 gigabytes) for each loudspeaker. For this reason, we have chosen a distributed client-server approach to software development. A Java based client program resides on a personal computer. This client takes care of the visualization and display of the acoustic information. The client program connects through the Internet to our computational servers based in Berkeley, California, which handle the numeric model computations.

In order to test our model and measurement facilities, we have constructed an ideal (in the classical physics sense) model loudspeaker, which consists of a well-behaved 3 inch cone woofer placed on the surface of a hard 10 inch radius sphere. Because of the simple geometry, it is possible to derive a solution to the complex pressure radiated by this loudspeaker using a simple recurrence relation and the sum of Spherical Hankel functions. Also, this geometry has the advantage of being rotationally symmetric, allowing for a great reduction in the number of empirical data points needed.

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We present both the theoretical results and the empirically measured data for this spherical loudspeaker model, and we use these results to discuss the accuracy of the far–field model.

2. Mathematical Formulation of a Spherical Loudspeaker

We start with the scalar wave equation that governs the acoustic propagation of small pressure disturbances in air,

$$\nabla^2 p(\mathbf{x},t) - \frac{1}{c^2}\frac{\partial^2 p(\mathbf{x},t)}{\partial t^2} = 0$$

where $\mathbf{x}$ is a point in $\mathbb{R}^3$, $p(\mathbf{x},t)$ is a linear acoustic pressure disturbance field, and t is time. If we assume time-harmonic phenomena, (no transient acoustic signals, only periodic vibrations)

$$p(\mathbf{x},t) = \mathrm{Real}\{\hat{p}(\mathbf{x})e^{-i\omega t}\}$$

we get the Helmholtz equation

$$\nabla^2 \hat{p}(\mathbf{x}) + k^2\,\hat{p}(\mathbf{x}) = 0$$

where f is frequency, k is the wavenumber, and λ is wavelength, $\lambda\, f = c$,c is the speed of sound in air, $k = \frac{2\pi}{\lambda}$, $k = \frac{\omega}{c}$. $\hat{p}$ is the (complex) time-harmonic acoustic pressure disturbance field, where $\hat{}$ denotes a complex number $a + i * b$, $a, b \in \mathbb{R}$ and $i = \sqrt{-1}$.

Since we want the solution to a radiation of a circular piston set on the surface of a sphere, we will use the spherical coordinates r, θ, ϕ, where r is the radial direction, θ is the latitude coordinate ranging from 0 at the north pole to π at the south pole (note this is a slightly different definition than in usual geography), and ϕ is the longitudinal coordinate ranging from $0 \longrightarrow 2\pi$.

The Laplacian operator ∇^2 in these spherical coordinates is

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin^2(\theta)}\frac{\partial}{\partial \theta}\left(\sin(\theta)\frac{\partial}{\partial \theta}\right) + \frac{1}{r^2\sin^2(\theta)}\frac{\partial^2}{\partial \phi^2}$$

It is shown in [3] that a solution to the Helmholtz equation in three dimensions can take the following form. Note that our problem has rotational symmetry in the longitudinal ϕ direction.

From this equation, we can derive a series solution for the pressure from a circular piston set in the side of a sphere:

$$\hat{p}(r,\theta) = \sum_{m=0}^{\infty} \hat{A}_m\, P_m(\cos\theta)\,\hat{h}_m(kr)$$

P_m are Legendre functions, h_m are spherical Hankel functions.

The spherical Hankel functions are defined as

$$\hat{h}_m(\xi) = j_m(\xi) + i n_m(\xi)$$

$$\hat{h}_m(\xi) = \frac{i^{-m}}{i\xi}\sum_{s=0}^{m}\frac{(m+s)!}{s!(m-s)!}\left(\frac{i}{2\xi}\right)^s e^{i\xi}$$

$$j_0(\xi) = \frac{\sin(\xi)}{\xi} \quad n_0(\xi) = -\frac{\cos(\xi)}{\xi}$$

(Note, that spherical Hankel functions are related to the Bessel functions of fractional order. The spherical Bessel function of the first kind is $j_n(z) = \sqrt{\frac{\pi}{2z}} J_{n+\frac{1}{2}}(z)$, and the spherical Bessel function of the second kind $y_n(z) = \sqrt{\frac{\pi}{2z}} Y_{n+\frac{1}{2}}(z)$, [1, page 437]. Note that in Morse's notation he uses $n_m = y_n$.)

The Legendre function is defined as [2]:

$$P_m(x) = \frac{1}{2^m n!} \frac{d^m}{dx^m}(x^2-1)^m, \quad n = 0, 1, 2 \ldots$$

or

$$P_m(x) = \sum_{s=0}^{[m/2]} \frac{(-1)^s (2m-2s)!}{2^m s!(m-s)!(n-2k)!} x^{m-2s}, \quad P_0(x) = 1, P_1(x) = x$$

where $[m/2]$ means the largest integer $\leq \frac{m}{2}$.

$$\hat{A}_m = \frac{\rho c U_m}{B_m} e^{-i\delta_m}$$

Our speaker piston is situated at the north pole of a sphere of radius a. If we define the vibration on our sphere to be

$$U(\theta) = \begin{cases} u_0, & \text{if } 0 \leq \theta \leq \theta_0 \\ 0, & \text{if } \theta_0 < \theta \leq \pi \end{cases}$$

$$U_m = \frac{1}{2} u_0 \left[P_{m-1}(\cos\theta_0) - P_{m+1}(\cos\theta_0) \right]$$

and for $m = 0, P_{-1} = 1$.

The following recurrence equations define the other parameters in the model:

$$(2m+1) B_m \sin\delta_m = (m+1) j_{m+1}(ka) - m j_{m-1}(ka)$$
$$(2m+1) B_m \cos\delta_m = m n_{m-1}(ka) - (m+1) n_{m+1}(ka)$$

$$\delta_m = \tan^{-1}\left(\frac{(m+1) j_{m+1}(ka) - m j_{m-1}(ka)}{m n_{m-1}(ka) - (m+1) n_{m+1}(ka)} \right)$$

From these equations , we can derive a numerical method to solve for the sound pressure from a spherical loudspeaker at any point in space.

For the physical constants, as average values of the speed of sound in normal temperature and pressure ranges, the speed of sound $c = 343 \frac{\text{meters}}{\text{second}}$ and ambient density of air $\rho = 1.2 \frac{\text{kg}}{\text{meter}^3}$ are used.

3. Comparison of the Theoretical Predictions and Empirically Measured Data

Figure 1. shows a comparison between the theoretical predictions derived in the previous section and actual data measured in an anechoic chamber. We built a model loudspeaker by mounting a standard 3 inch cone woofer flush on the surface of a hard plastic sphere with a 10 inch radius. This model loudspeaker was then mounted on a rotation device in an anechoic chamber. A multi-timescale FFT based measurement system was used in conjunction with a signal consisting of pink noise. The system gives approximately 1/24th octave resolution to 20kHz. The spherical loudspeaker was rotated and measured every degree, for a total of 360 measurements. The magnitude of the measured complex pressure experimental data is plotted in red, as a continuous line for easy identification. The theoretical prediction was also calculated every degree, and the magnitude of the complex pressure is plotted as a continuous blue line. The radius of the piston for the theoretical prediction was chosen to be 1.25 inches, see the discussion below for the reasons. The scale of the polar pattern axis is linear, and the radial axis scale ranges between 1 and zero. Each individual polar pattern trace is normalized, with the largest magnitude equal to 1.

In general, the theoretical predictions match the experimental results very closely. In the theoretical model, the piston in the sphere is assumed to be convex, while the experimental setup uses the standard inverted cone loudspeaker. This loudspeaker is most likely not radiating exactly equally for the range of frequencies studied. In fact, if the piston radius is chosen as 1.5 inches (which assumes the suspension contributes to the sound pressure radiated), the agreement between the low frequency theoretical predictions and the measured data is slightly better, while at the high frequencies it is slightly worse. Conversely, if the radius is chosen as 1 inch, the theoretical prediction matches slightly better at the high frequencies, but slightly worse at the low frequencies. At the higher frequencies, the suspension in probably decoupled from the radiating cone, so a smaller area is vibrating. The differences in the frequency range 300-600 Hz might be caused by reflection and diffraction off of the rotation machinery. Since this machinery was designed to rotate a loudspeaker up to 2000 lbs. in full three-dimensional rotation, it is not as small as ideally would be the case.

4. A Far–Field Model of Loudspeaker Interaction

We define the far–field polar pattern of a loudspeaker to be the complex pressure $\hat{p}^{\infty}$ measured on the surface of a sphere of radius r_{f} where $r_{\mathrm{f}} \ggg a$, where a is the characteristic length scale of the loudspeaker (or radius in the case of a spherical loudspeaker). For theoretical predictions, $\hat{p}^{\infty}$ can be derived from the limit as $r_{\mathrm{f}} \to \infty$. For empirical measurements, r_{f} is the largest practical distance based on the size of the anechoic chamber being used, signal to noise ratio, etc.

$$\hat{p}_{\mathrm{sum}} = \sum_{\beta=1}^{\#\text{of speakers}} \hat{p}^{\infty}_{\beta}(\theta,\phi)\, \frac{1}{r_{\beta}}\, e^{i\,k\,r_{\beta}}$$

where r_{β} is the distance from each loudspeaker to the position being calculated, and $\hat{p}^{\infty}_{\beta}(\theta,\phi)$ is the complex polar patter of each individual loudspeaker.

Note that this model does not model the acoustic waves that scatter off of the other loudspeakers. At first glance this would appear to be a serious deficiency. However, modern high-powered professional loudspeakers are usually designed to be highly directional, and thus the scattered field is very small compared to the direct field. We find the results of this far field model of interaction to be surprisingly accurate when used with high resolution empirically measured polar patterns. Due to the simplicity of the model, the model is only as accurate as the polar patterns used. If phase data is ignored, or if a coarse frequency and spatial measurement grid is used, the results will be inaccurate. Essentially, we are using a simple theoretical model with high-resolution empirical data to provide accurate results in an interactive software environment. Currently, advanced numerical models such as boundary integral or finite element methods require too much computation for interactive design software. Since computational power is rapidly increasing, with no end in sight, soon advanced numerical models could be utilized that would capture the scattering effects of multiple loudspeakers.

During the presentation of this paper, we will demonstrate our current client-server based MAPP($^{\mathrm{tm}}$) far–field loudspeaker interaction software. We will show the prediction of two spherical loudspeakers placed in a parallel array, as well as empirical results from two identical spherical loudspeakers measured in our anechoic chamber. Time permitting, we will show results from measured loudspeakers, in realistic arrays.

References

[1] M. Abramowitz and I. A. Stegun, eds., *Handbook of Mathematical Functions With Formulas, Graphs, and Mathematical Tables*, Dover, New York, N.Y., 1972.

[2] N. Lebedev, *Special Functions and Their Applications*, Dover, New York, 1972.

[3] P. M. Morse and K. U. Ingard, *Theoretical Acoustics*, Princeton University, Princeton, NJ, 1986.

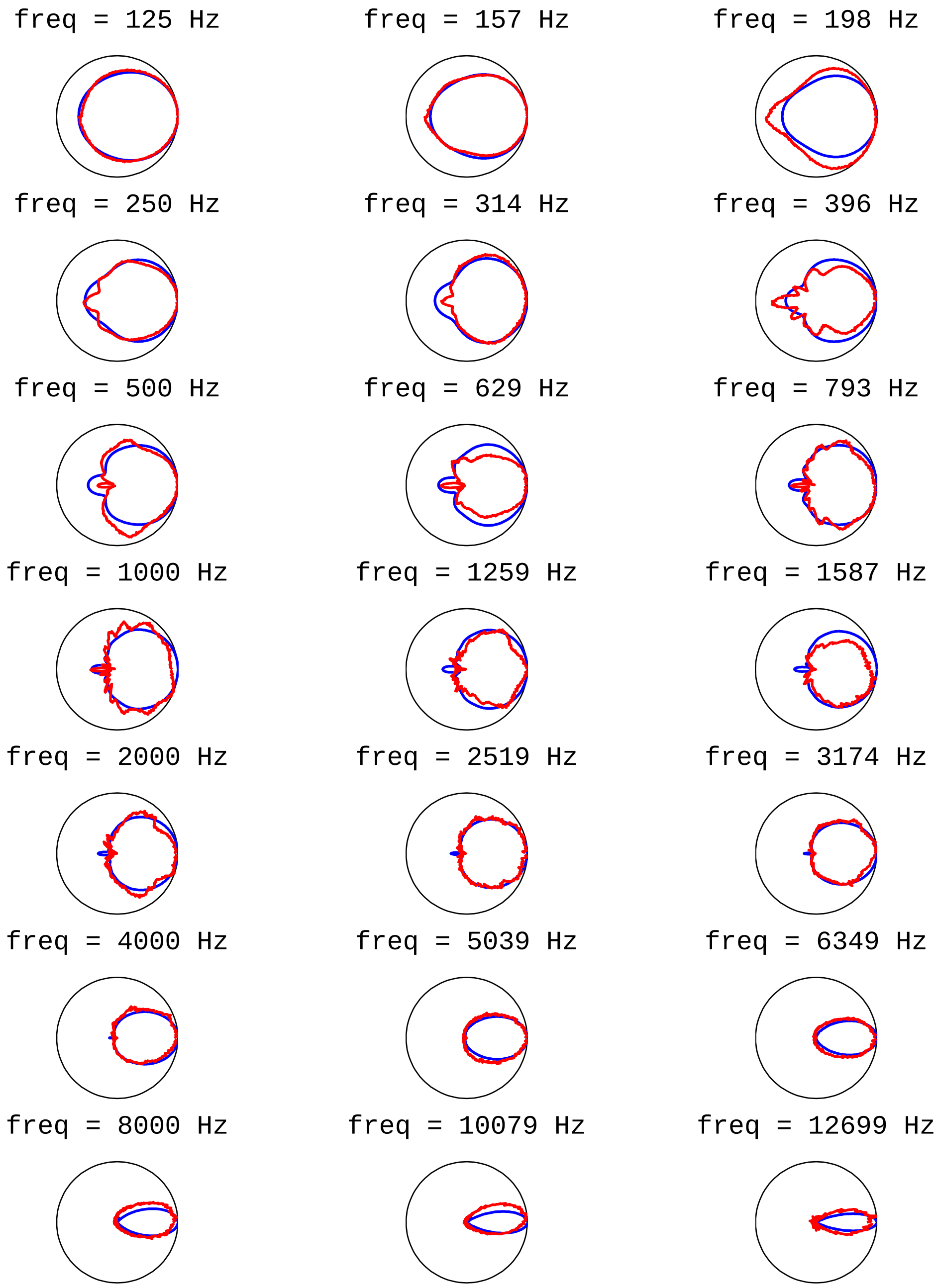

FIGURE 1. Polar patterns of a 3 inch woofer set in a 10 inch radius spherical enclosure. The scale is linear, and shows the magnitude of the complex pressure. The blue line shows the theoretical prediction, the red line shows the measured data.