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Carrying Capacity: A Model with Logistically Varying Limits

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ABSTRACT

We introduce an extension to the widely-used logistic model of growth to a limit that in turn allows for a sigmoidally increasing carrying capacity, that is, the invention and diffusion of technologies which lift the limit. We study the effect of this dynamic carrying capacity on the trajectories of simple growth models, and we use the new model to re-analyze two actual cases of the growth of human populations. English and Japanese examples with two pulses, or one change in limit, appear to verify the model.

1. Simple Growth Model with a Logistically Increasing Carrying Capacity

Finding convincing or widely agreed upon estimates and models for carrying capacity, especially for human populations, is difficult. Cohen [1] archives 26 different attempts from a variety of fields, few of which agree on much. Most do agree that changes in technology affect the carrying capacity of a system [2]. Evidently, new technologies affect how resources are consumed, and thus if carrying capacity depends on the availability of that resource, the value of the carrying capacity would change. For example, raising yields has allowed the developed world to support an increasing population while cropping a decreasing amount of land [3]. For these reasons, models of growth for human systems based on *fixed* resource limits or a single, unchanging carrying capacity are unrealistic.

Importantly, although a new technology may offer a significant increase in efficiency, it does not spread instantly, but instead is adopted at a changing rate [4]. These adoption processes are often well modeled by a logistic [5]. First, the rate of adoption is slow, as a new technology must struggle to replace a mature one. The rate of adoption increases, usually exponentially until physical or other limits slow the adoption. Adoption is a kind of "social epidemic." Moreover, learning itself can be sigmoidal: we proceed along "learning curves," improving our performance with experience [6]. Indeed, Schelling [7] portrays an array of logistically developing and diffusing social mechanisms. Finally, technological innovations do not usually distribute themselves evenly through time, but instead cluster in spurts or "innovation waves" [8]. For these reasons, we formulate a model where the carrying

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capacity $\boldsymbol{\kappa}$ of a system increases dynamically, but in a distinct pulse. Because the simple logistic is well-suited as a model for learning as well as diffusion phenomena, we model the carrying capacity $\boldsymbol{\kappa}(t)$ itself as a logistic function of time. Below we develop the mathematics of this proposal and, finally, test it against the population histories of England and Japan.

We can mathematically model a dynamic carrying capacity by extending the logistic differential equation

$$\frac{dP(t)}{dt} = \alpha P(t)\left(1 - \frac{P(t)}{\boldsymbol{\kappa}}\right). \tag{1}$$

Here, $P(t)$ is the population, α is the exponential growth rate parameter, and $\boldsymbol{\kappa}$ is the saturation or ceiling value of the sigmoidal logistic curve. We often replace α with a parameter we call the "characteristic duration," or $\Delta t = \frac{\ln(81)}{\alpha}$, where Δt refers to the time for the logistic curve to grow from 10% to 90% of saturation $\boldsymbol{\kappa}$. Specifically, we replace the constant κ in equation (1) with a function $\boldsymbol{\kappa}(t)$:

$$\frac{dP(t)}{dt} = \alpha P(t)\left(1 - \frac{P(t)}{\boldsymbol{\kappa}(t)}\right). \tag{2}$$

The addition of the dynamic carrying capacity $\boldsymbol{\kappa}(t)$ increases the complexity of the behavior of the model, as $\boldsymbol{\kappa}(t)$ can be any function, and numerous extensions have been proposed and studied over the years. Banks [9] describes models where $\boldsymbol{\kappa}(t)$ has been varied sinusoidally, exponentially and linearly. Coleman [10] has studied general forms of (2) and determined global mathematical properties. Cohen [11] presents a model similar to (2) in his discussion of global human carrying capacity where the carrying capacity $\boldsymbol{\kappa}(t)$ is itself a function of the population $P(t)$.

As described, the adoption of new technologies is well modeled by the logistic model. For this reason, we will study an extension to (2) where the carrying capacity $\boldsymbol{\kappa}(t)$ is modeled as a logistic, a logistic inside a logistic:

$$\frac{d\boldsymbol{\kappa}(t)}{dt} = \alpha_\kappa \boldsymbol{\kappa}(t)\left(1 - \frac{\boldsymbol{\kappa}(t)}{\kappa_\kappa}\right). \tag{3}$$

Mathematically, $\boldsymbol{\kappa}(t)$ is identical to the $P(t)$ in equation (1). However, this model (equation (3)) assumes $\boldsymbol{\kappa}(t)$ starts at zero, which is unrealistic for most technologies. A new technology starts with some carrying capacity or "initial potential" that is nonzero.

A modification to (3)

$$\frac{d\boldsymbol{\kappa}(t)}{dt} = \alpha_\kappa(\boldsymbol{\kappa}(t) - \kappa_1)\left(1 - \frac{(\boldsymbol{\kappa}(t) - \kappa_1)}{\kappa_2}\right) \tag{4}$$

describes a logistic that increases sigmoidally between an initial value κ_1 and a final value κ_2, as the analytic solution makes clear:

$$\boldsymbol{\kappa}(t) = \kappa_1 + \frac{\kappa_2}{1 + \exp\left(-\alpha_\kappa(t - t_{m_\kappa})\right)}. \tag{5}$$

Here, t_{m_κ} is the midpoint (or inflection point) of the carrying capacity logistic (equation (5)).

The advantage of this previously unstudied model is that it allows for the often seen phenomenon of "bi-logistic" growth [12], where a growth trajectory nearing the initial carrying capacity or ceiling starts growing again to a second, higher,

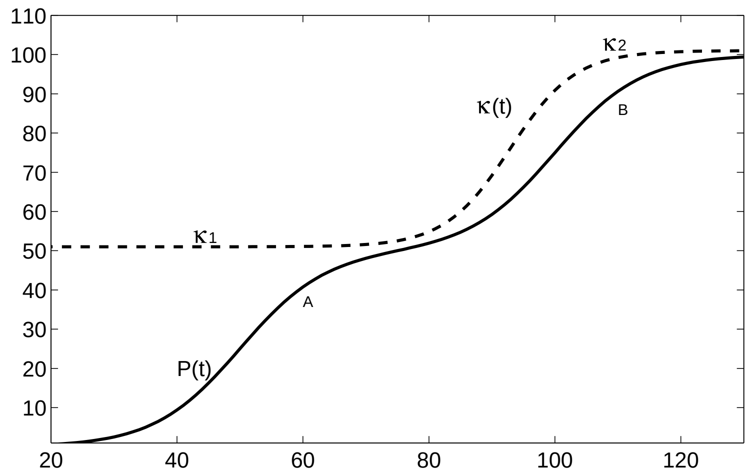

FIGURE 1. Bi-logistic Growth From Logistically Increasing Carrying Capacity $\boldsymbol{\kappa}(t)$.

carrying capacity. The rise in airplane performance with first piston engines and then jet engines provides a familiar technological example.

A hypothetical example shows visually how our model can describe bi-logistic growth. Figure 1 shows a generated growth trajectory $P(t)$ for the given logistic carrying capacity $\boldsymbol{\kappa}(t)$. The pulses A and B in Figure 1 were produced by the logistic rise of $\boldsymbol{\kappa}(t)$ from κ_1 to κ_2 depicted by the solid curve labeled $\boldsymbol{\kappa}(t)$.

By its production of the bi-logistic as a special case, the model with a logistically changing capacity demonstrates it is different in outcome as well as logic from a simple logistic with a slower rate. The simplicity and clarity with which the model generates the often seen bi-logistic growth pattern suggests its utility for understanding the growth of populations and for abstracting and formalizing the notion of a dynamic carrying capacity.

One challenging aspect of our model is determining the parameters that generate a good fit with observed data. In order to specify completely our model, we must determine the six parameters ($P(0), \alpha$,κ_1, κ_2, α_κ, and t_{m_κ}). For the following two empirical examples, a numerical Monte-Carlo approach was used. Specifically, a simulated annealing method of non-linear optimization [13] was used to find the parameters that minimized the least-squared residual error between the numerically integrated $P(t)$ and the population data. A standard 4th-order Runge-Kutta algorithm was used to numerically integrate the model. [14].

2. Dynamic Carrying Capacity and Human Population - A Test

In Section 1, we emphasized that exploring, inventive humanity exemplified the lifting of carrying capacity. Through the invention and diffusion of technology, humans alter and expand their niche and violate population forecasts. In the 1920's, Using a single logistic, Pearl [15] estimated the globe could support two billion people, while today about six billion dwell here [1].

One of the greatest technological shifts was the industrial revolution, which changed societies from primarily agrarian to manufacturing and services. The first

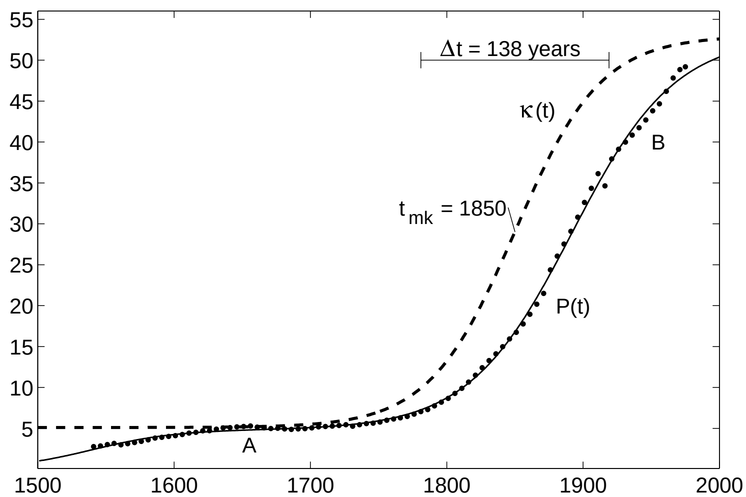

FIGURE 2. Population of England 1541-1975. Source of Data [16].

society to do so was England. Figure 2 shows the population of England from 1541-1975 fit with the logistic carrying capacity model (2). The early English, islanders conceptually similar to the bacteria in a petri dish, could not directly expand their territory to support more people. In fact, they had already cleared a large fraction of the land for crops and animal husbandry by Roman times. English population shows a slow rise (pulse "A"), leveling off around 5 million people (κ_1) in 1650. Perhaps sensing their local limit, the English were actively colonizing abroad during the 17th and 18th centuries and exporting population. The population remained rather level until nearly 1800. Meanwhile, another sigmoidal pulse ("B") of 50 million (κ_2) had begun, bringing England close to its current population. Faster and cheaper transport, new energy sources, and other factors made it possible for ten times more English to eat in the same dish, at the outset in large part by exchanging manufactured cloth for foreign grain. The model shows the hypothetical increase in the carrying capacity $\boldsymbol{\kappa}(t)$ is centered in 1850 with a characteristic duration of 138 years. The parameters of the model provide a quantitative definition of the Industrial Revolution for England and locate it precisely in time.

Japan, another space-constrained island nation, provides an even starker example of the effects of technological change on a population (Figure 3). From the 12th to the middle of the 19th century, Japan was primarily a feudal society, with little industry or foreign trade. The population grew sigmoidally (from an initial base of 5 million) to a level (κ_1) of 35 million. In the latter half of the 19th century the Meiji restoration embarked Japan on a rapid journey of industrialization and modernization, with an explosive increase in population. Changes in agriculture, transport, and other industries allowed Japan to provide for about 90 million more people (κ_2).

The parameters expedite a comparison of the English and Japanese Industrial Revolutions. England crossed 1% in 1725 and 10% in 1808, while Japan crossed 1% in 1832 and 10% in 1869. The Japanese Industrial Revolution centered in 1908, almost 60 years later than the English, and had a characteristic duration

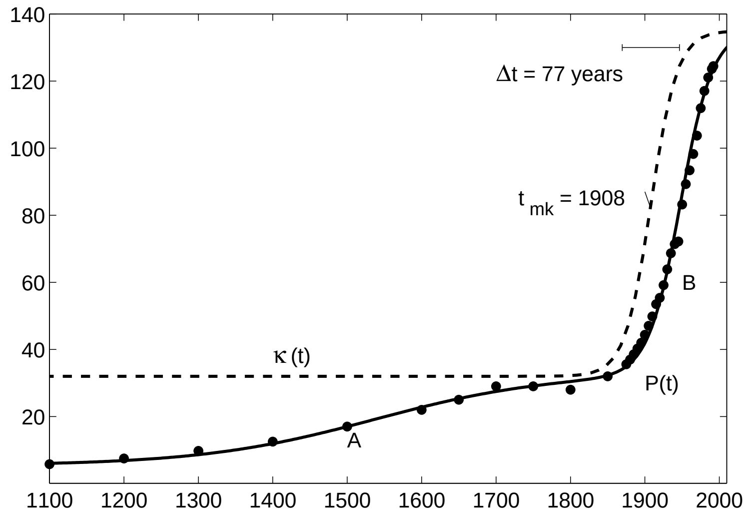

FIGURE 3. Population of Japan 1100-1992. Source of Data [18, 19].

of 77 years, almost twice as steep. This steepening or shortening of the diffusion time is consistent with numerous studies showing that technologies spread faster in countries that begin to adopt technologies later [17]. The countries that adopt technologies later borrow much and need to experiment less to make the system work.

Continuing to probe the meaning of the estimates of the parameters, one must ask what sets κ_1 and κ_2. Conceivably the κ_1 of five million in England and thirty five million in Japan were set by the physical limits of food. At least the history of bread riots in Nottingham in the early nineteenth century encourages that belief. The present limit κ_2 on population in England and Japan, however, cannot conceivably be set by food. Instead the current stabilization reflects choices about family size as well as access to resources [20]. Nevertheless, we doubt that the Japanese or English would again multiply by 3 or 10 as they have just done without comparably significant technological changes that maintain or enhance the quality of life. More generally, answering the question of future human numbers will require more than estimates of carrying capacity.

3. Conclusion

More than a century of experience proves that the simple logistic that describes, say, the growth of bacteria confined in a petri dish with a fixed amount of nutrition will not fit the multiplication of humanity. We propose that logistic growth within a dynamic carrying capacity that itself rises logistically fits better. The model retains much of the elegance of the logistic as well as its familiar form. With this equation to fit to and summarize the course of the inventive and exploring human population, the task becomes one of understanding what sets and how fast and far carrying capacity rises, and perhaps from that understanding, anticipating the new levels.

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